To determine the spectral characteristics most exactly, the amplitude–frequency characteristic (AFC) with a clearly pronounced discrete spectrum is necessary. The width of a resonant peak depends on the figure of merit of the system, which is determined by its dissipative properties. The introduction of the inertial switching on into such a system varies its spectrum, which can be used to reveal switching on itself. The accuracy of determining the resonant frequency depends on the width of the AFC resonant peak (the accuracy is inversely proportional to the figure of merit). Therefore, on the one hand, it is necessary to increase the figure of merit for the most exact determination of the spectral characteristics, and on the other hand, to decrease it in order not to miss the resonance under study. In this work, we assume use of the nonlinearity of the AFC nanoresonator situated under the effect of the ac electric field with the purpose of more stable determination of its varying resonant properties.

New nanomaterials fabricated recently and the technologies of their use promote the development of new, in principle, nanoelectromechanical systems, particularly nanoresonators. Reviews of the modern state of such systems and their potential applications are given in [1, 2]. One of the most widespread applications of resonators is their use as mass detectors. These systems include a thin film, the crystal, or the cantilever, which oscillates with a frequency up to several hundred megahertz. Due to the adherence of the particle (molecule, atom) to a flexible surface, its resonant frequency changes. This effect makes it possible to determine the mass of the adhered particle.

A serious disadvantage in using the graphene-based resonator is its low figure of merit (of about 100) [2, 3]. The latter is caused by the “Joule” dissipation, i.e., by the transformation of electric energy into thermal energy because of the appearance of eddy currents in the graphene layer itself [4, 5].

In this article, we consider a new, in principle, possibility of using such a resonator, which makes it possible to increase the measurement accuracy of the eigenfrequency at a low figure of merit of the oscillatory system. The resonator based on the graphene layer is considered as the electromechanical oscillatory system, in which mechanical oscillations are excited by the ac electric field in the space between the graphene layer and the conducting surface. Such a system is a capacitor with the capacity depending on the transverse deformation of the graphene layer. The electric field is induced by the external ac voltage source. In contrast with the use of the linear AFC, we propose to take into account the nonlinear effects that follow oscillations in the electric field. They lead to a “soft”

**Oscillation Stop as a Way to Determine Spectral Characteristics of a Graphene Resonator**


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Abstract—A nanoresonator based on a graphene layer is investigated as an electromechanical oscillatory system. Mechanical oscillations are excited in it by a high-frequency alternating electric field. A nanoresonator is considered as a capacitor with kinematically varying capacity of the determined transverse deformation of the graphene layer as one of its plates. In the case of small ratios of energy accumulated in a capacitor to the amplitude of energy of mechanical oscillations and the time constant of the capacitor charge to the period of free oscillations, excitation of both common and parametric resonances is possible. It is shown that upon decreasing the external frequency lower than the half-frequency of free oscillations, the cessation of forced oscillations of the nanolayer is observed. This makes it possible to determine more reliably the variations in the intrinsic frequency of the nanoresonator upon deposition of a nanoparticle on it.

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AFC with the cessation of oscillation. The measurement of the cessation frequency is possible with a higher accuracy compared with finding the amplitude maximum of the linear AFC.

The graphene layer and the conducting surface are the plates of the capacitor, which is connected to the ac voltage source. The graphene layer bends under the effect of mutual resistance forces between the capacitor plates. This leads to varying the capacitor capacity depending on deflection. In the first approximation, we assume that capacity depends on the deflection of graphene layer \( x(t) \)

\[
C = \epsilon \varepsilon_0 \frac{S}{d_0 - x(t)},
\]

where \( S \) is the plate area and \( d_0 \) is the distance between the graphene layer and the conducting surface in the absence of the electric field.

The motions of the electromechanical system are described by the set of mutually related equations

\[
m\ddot{x} - \frac{Q^2}{2C^2} \frac{dC}{dx} + c_q x = 0, \quad \dot{Q} + \frac{1}{RC} Q = \frac{U}{R} \sin \omega t,
\]

where \( m \) is the mass of the graphene layer, \( c_q \) is its bending stiffness, \( Q \) is the capacitor charge, and \( R \) is the resistance of the electric circuit supplying the ac voltage \( U \sin \omega t \) to the capacitor. The first equation of set (2) describes mechanical oscillations of the graphene layer (capacitor plates) under the effect of the electric force of mutual attraction. The second equation of set (2) is the equation of the voltage balance in the electric circuit.

Let us introduce dimensionless variables \( \tau = \lambda t \) and \( q = \frac{Q}{Q_0} \), where \( \lambda = \sqrt{\frac{c_q m}{\varepsilon}} \) is the eigenfrequency of bending oscillations of the graphene layer and \( Q_0 = C_0 U_0 \) is the starting constant charge of the capacitor appearing under the effect of the supplied dc voltage \( U_0 \), \( \xi = \frac{x}{d_0} \).

As a result, we derive a set of equations with the “main” parameters, where the point denotes the derivative by the new “dimensionless” time \( \tau \), \( \Omega = \frac{\omega}{\lambda} \):

\[
\ddot{\xi} - \alpha q^2 \xi = 0, \quad \dot{q} + \kappa (1 - \xi) q = \tilde{u} \kappa \sin \Omega \tau.
\]

Here, parameters

\[
\alpha = \frac{C_0 U_0^2}{2m\lambda^2 d_0^2}, \quad \kappa = \frac{1}{RC_0 \lambda}, \quad \tilde{u} = \frac{\tilde{U}}{U_0}.
\]

The physical meaning of the parameters introduced is as follows: \( \alpha \) is the ratio of the potential (electrical) energy of the capacitor with the initial accumulated charge \( Q_0 = C_0 U_0 \) to the kinetic energy amplitude during harmonic oscillations of the plate having mass \( m \) with intrinsic frequency \( \lambda \) and amplitude \( d_0 \), \( \kappa \) is the ratio of the period of free mechanical oscillations of the capacitor plate to the damping time, or decreasing the initial charge of the capacity connected to resistance \( R \), and \( \tilde{u} = \frac{U}{U_0} \) is the scaled amplitude of the external harmonic voltage.

The amplitude of plate oscillations far from resonance or at a low applied ac voltage is small (small \( \tilde{u} \)). In this case, we can consider that the capacity is independent of the deflection and Eqs. (3) are linear. Forced oscillations of the graphene layer with frequency \( 2\Omega \) occur in this case.

Let us consider the equilibrium of the graphene layer under the effect of a constant electric voltage \( \Omega = 0 \). It is evident that the undeformed state of the graphene layer is not the equilibrium position in the presence of the dc electric field. Two equilibrium positions can occur, notably, a stable one with a smaller deformation and an unstable one with a larger deformation; or, depending on the applied voltage, the equilibrium position may be completely absent.

The account of the influence of the graphene layer on the capacitor capacity leads to three main conclusions. First, the interaction force is equivalent to the presence of the elastic basis with nonlinear negative elasticity. Second, the external field excites the forced oscillations with a doubled frequency relative to the external effect frequency (electric voltage). Third, excitation of parametric oscillations is possible.

Resonant modes are possible near \( \Omega = 0.5 \) (the coincidence of the common resonance with the second parametric one) and at \( \Omega = 1 \) (main parametric resonance).

We will seek the approximated solution of nonlinear set (3) under the assumption of smallness of \( \alpha = \varepsilon \tilde{\alpha} \) and a large value of \( \kappa = \frac{\tilde{\kappa}}{\varepsilon} \). The solution of set (3) can be found by a method similar to the van der Pol method. We seek variable \( \xi \) in the form of a quasi-harmonic function with “slow” coefficients \( a_s \) and \( a_c \) at harmonics and with obligatory holding of slowly varying constant component \( a_0 \):

\[
\xi(\tau) = a_0(\varepsilon \tau) + a_s(\varepsilon \tau) \sin \tau + a_c(\varepsilon \tau) \cos \tau,
\]

\[
\dot{\xi}(\tau) = a_s(\varepsilon \tau) \cos \tau - a_c(\varepsilon \tau) \sin \tau.
\]