Multiscale simulation of friction with normal oscillations in the method of reduction of dimensionality

R. Heise

Berlin University of Technology, Berlin, 10623, Germany

In the framework of the method of reduction of dimensionality, the dependence of the kinetic coefficient of friction of elastomers with linear rheology on normal loads with a static and an oscillatory part has been examined. It is shown that the sinusoidal excitation leads to a reduction of the coefficient of friction by up to 20% for a sliding velocity range of two orders of magnitude. The largest reduction of the coefficient of friction occurs at a velocity, which is proportional to the characteristic wave length of the surface and the frequency of the acting normal force.

Keywords: coefficient of friction, elastomer, active control, sinusoidal excitation

DOI: 10.1134/S1029959912030125

1. Introduction

The computation of the frictional force between an elastomer and a rigid surface still poses a complicated problem in contact mechanics. Due to the fact that several length and time scales are involved the complexity of the problem is considerable. To cope with the multiscale spatial character the method of reduction of dimensionality was developed in [1] (for a detailed treatment [2]). The contact between three-dimensional (visco)elastic bodies is substituted by the contact of two rough one-dimensional profiles. The interesting contact mechanical properties such as real contact area [1], real contact length [3], adhesion [4] and frictional force for materials with simple rheology [5] can be computed according to rules after the reduction from three to one dimension. In the studied problem, a further ingredient is a time dependent excitation of the elastomer, which establishes another time scale in addition to the range of genuine relaxation times characterizing elastomers. This excitation is introduced to numerically investigate the fact that friction is largely reduced by mechanical oscillations [6]. Simple models concerning stick-slip motion [7] showed a reduction of the coefficient of friction. A thorough understanding of the mechanisms that reduces the coefficient of friction is still lacking. In this study, we perform a numerical experiment based on the method of reduction of dimensionality [2, 8] and the usage of a hierarchical memory [5].

2. Modeling

The set up of the considered model is the following: A soft surface with properties of an elastomer is pressed against a rigid counter body which is sliding at a constant velocity. Within the method of reduction of dimensionality, both bodies are mapped to a one-dimensional analogue to a two-dimensional surface. The one-dimensional substitute surfaces consist of two sets of independent points which do not interact along the chain.

For the sake of simplicity, the original surface is assumed to have a constant power spectrum within a narrow wave vector band

\[ C_{2D}(q) = c_0 \left( \frac{q}{q_f} \right)^{-\beta} , \quad q_l \leq q \leq q_f , \]

with \( c_0 = 10^{-27} \text{m}^{-4} \), \( q_l = 10^5 \text{m}^{-1} \), \( q_f = 2 \cdot 10^5 \text{m}^{-1} \), \( \beta = 0 \).

Hence, the root mean square of the surface is \( \text{rms} = \sqrt{\langle h^2 \rangle} = 9.42 \text{ nm} \) and its mean slope \( \langle \nabla z \rangle = \sqrt{\langle (\nabla h)^2 \rangle} = 0.00115 \). This corresponds to a characteristic wave length of \( \lambda \approx 2\pi \text{rms}/\langle \nabla z \rangle \approx 51 \mu \text{m} \).

The one-dimensional substitute surfaces are generated using the spectral density defined according to the rule [1, 2]...
\[ C_{1D}(q) = \pi q C_{2D}(q) \]  
with a random phase \( \varphi \):

\[ h_{1D}(x) = \int q \pi C_{2D}(q) \exp[iqx + i\varphi] dq. \]  

It possesses the same root mean square of points and mean slope as the original one. The number of points along the one-dimensional line is chosen to be \( N = 2^{17} \). The large number of points ensures good statistics on the contacts.

On the soft body, an external normal force is exerted. The overall normal force consists of a static and a sinusoidally oscillating one

\[ F_n(t) = F_{n0}(1 + \chi \sin(2\pi ft)) \]  
with frequency \( f \), average value \( F_{n0} = 1.9 \) mN and relative amplitude \( \chi = 0.99 \).

This external force is matched by the sum of forces at all points in contact. These contact forces are calculated according to [2]

\[ F_c(t) = 4\pi x \int G(t - t') v(t') dt', \]  
where \( dx \) is the spacing between the discrete points and \( v \) is their deformation velocity. The memory function of the elastomer \( G(t) \) is modeled in the following form [2]:

\[ G(t) = G_o + G_1 \tau_1 \int \tau^{-\mu} e^{-t/\tau} d\tau \]  
with \( G_o = 1 \) MPa, \( G_1 = 1 \) GPa, \( \tau_1 = 10^{-2} \) s, \( \tau_2 = 10^2 \) s and \( s = 2 \). Hence, it contains a wide range of relaxation times from 10 milliseconds up to 100 seconds. The numerical evaluation of the integral over the deformation history is rather time consuming. Therefore, the integral in Eq. (5) is computed at exponentially distributed supports following the idea of a hierarchical memory put forward in [5].

Points are considered to be in contact if the rigid surface is pressed into soft surface and the force acting is positive. The second condition avoids adhesive effects between the surfaces. The overall distance of the interacting bodies is adapted in such a way that the overall force equals the normal force in this time step. Points on the deformable surface are set to the rigid surface coordinate if they are in contact. Points without contact relax.

The force acting on the elastomer and the rigid counter body convoluted with the slope of the rigid surface at the points in contact is defined as the tangential frictional force. The coefficient of friction is the ratio between the sum of all frictional forces and the acting time averaged normal force. The sliding speed is varied in the interval from \( 2 \times 10^{-8} \) to \( 2 \times 10^{-3} \) m/s.

An analytic estimate for the spectral velocity dependence of the normalized coefficient of friction may be obtained from considerations of the energy dissipation in an elastomer [3, 9, 10]. This estimate is given by

\[ \frac{\mu}{\sqrt{\nu}} = \frac{\dot{\gamma}^*}{|\dot{\gamma}(\omega)|} \left[ 1 + \frac{\dot{\gamma}^*(\omega)}{\dot{\gamma}^*(\omega)} \right]^{-1/2}, \]  
where \( \dot{\gamma}(\omega) = \dot{\gamma}^*(\omega) + i\dot{\gamma}^*(\omega) \) is the complex shear modulus in the frequency domain. Evaluating this expression for the proposed elastomer model (5), the real and imaginary parts of the shear modulus read

\[ \hat{G}^\prime(\omega) = G_o + G_1 \int_{\tau_1}^{\tau_2} \frac{\omega^2 \tau^2}{\tau + 1 + \omega^2 \tau} g(\tau) d\tau = G_o + G_i \omega \tau_i \left( \arctg(\omega \tau_i) - \arctg(\omega \tau_i) \right), \]  
\[ \hat{G}^*(\omega) = G_i \int_{\tau_1}^{\tau_2} \frac{\omega \tau}{\tau + 1 + \omega^2 \tau} g(\tau) d\tau = \frac{1}{2} G_i \omega \tau_i \ln \left( \frac{\tau_i}{\tau_i} + 1 + \omega^2 \tau_i^2 \right) \]  

The normalized coefficient of friction rises from low values to values close to unity at frequencies of approximately \( G_i / (G_i \tau_i \ln(\tau_i / \tau_i)) \) and falls down to almost zero value at frequencies of about \( 1/(2 \tau_i) \). In order to compare the estimate to the numerical simulation the sliding velocity is linked to the angular frequency via the characteristic stochastic values of the surface \( v_s = \sqrt{\nu \omega} \).

3. Results and discussion

It is apparent that the built model provides good agreement between the numerical values of the coefficient of friction and the analytical estimate (Fig. 1).

From Fig. 2 it is obvious that the characteristic value of the highest reduction is progressing to higher velocities as the excitation frequency rises. The velocity at which the largest reduction occurs is related to the frequency

\[ v = f \lambda \]  
with a proportionality constant close to unity. The reduction is most apparent in the medium sliding velocity range and reaches about approximately two orders of magnitude in velocity.

Here, the dissipation of energy overtakes the storage of energy as can be seen from the estimate of the loss and

![Fig. 1. Coefficient of friction vs. sliding velocity. The solid line provides the analytic estimate based on the energy dissipation in an elastomer. The dotted line shows the computation result for the discrete one-dimensional model without external oscillatory excitation](image-url)