Adhesive properties of contacts between elastic bodies with randomly rough self-affine surfaces: A simulation with the method of reduction of dimensionality

V.L. Popov and A.E. Filippov*1

Berlin University of Technology, Berlin, 10623, Germany
1 Donetsk Institute for Physics and Engineering NASU, Donetsk, 83114, Ukraine

Adhesive properties (adhesion force and adhesion coefficient) of contacts between elastic bodies with rough surfaces have been investigated and adhesion maps constructed showing the dependence of adhesive properties on the roughness, rms slope of the surface, elastic modulus, surface energy and fractal dimension. Simulations have been carried out in the frame of the method of reduction of dimensionality.

Keywords: contact, adhesion, rough surfaces, self-affine surfaces, adhesion coefficient, pressure sensitive adhesion

DOI: 10.1134/S1029959912030149

1. Introduction

The roughness of surfaces has a great influence on many physical phenomena, such as friction, wear, sealing, adhesion, as well as electrical and thermal conductivity [1–4]. Bowden and Tabor [1] were the first to realize the importance of the surface roughness of bodies in contact. The main qualitative understanding of contact properties of rough surfaces — including self-affine surfaces — is due to the works by Archard [5]. Greenwood and Williamson published a theory of contacts with rough surfaces [6], which became the most influential model for many years. The most important findings of Archard as well as Greenwood and Williamson were that the main effect of the roughness is due to the increase in the number of microcontacts with normal force while the average size and the loading conditions in individual microcontacts change only slowly. While the surfaces considered by Archard were partially multiscale self-affine surfaces, they have not been “randomly rough.” The surfaces considered by Greenwood and Williamson were on the contrary randomly rough but neither multiscale nor self-affine. Further generalization of the contact mechanics and its application to random self-affine surfaces is due to a number of researchers. Main contributions are the Persson theory [4] as well as numerical simulations by Hyun and Robbins [7] and Campana and M"{u}ser [8]. The main interest was focused over several years on the real contact area. Recently, the contact stiffness was studied in detail [9–11]. In the present paper we will deal with adhesive contacts between elastic bodies with fractal surfaces. The physical foundations of the underlying theory have been established by Griffith [12] in his theory of adhesive crack. This theory has been microscopically supported by Prandtl [13] and applied to contact problems between an elastic sphere and a halfspace by Johnson, Kendall and Roberts [14]. Some generalizations of this theory can be found in [15]. In the last decade, adhesion of structured and rough surfaces became a hot topic, due to the interest in the adhesion mechanism in living systems [16, 17]. Adhesion is further considered to be one of contributions to the friction force [18].

Inspite of theoretical and practical importance of the adhesive contact problem, no numerical simulations of contact between bodies with fractal surfaces were done until now, because of the complexity of this problem for numerical implementation. In the present paper we close this gap by direct numerical simulations of adhesion properties between stochastically rough, self-affine surfaces. For this sake, we use the method of reduction of dimensionality. This method was proposed in [19] and allows us to substitute a real three-dimensional contact with a contact with a one-dimensional elastic foundation. This has been initially proposed for normal contact between cylindrical and parabolic indenters and an elastic half-space [19] and then extended to randomly rough (but not fractal) surfaces [20, 21].

* Corresponding author
Prof. Alexander E. Filippov, e-mail: filippov_ae@yahoo.com

© Pleiades Publishing, Ltd., 2012. All rights reserved.
Distributed worldwide by Springer.
[22] proof has been provided that the method of reduction of dimensionality gives exact results for contacts of arbitrary bodies of revolution, both with and without adhesion. Quite recently, a comparison of direct three-dimensional calculations and calculations with the method of reduction of dimensionality have shown that it is applicable to self-affine fractal surfaces as well, in the range of fractal dimensions from 2 to 3 [23]. In the meantime, the method has been successfully applied to simulation of frictional force between a rigid rough surface and an elastomer [24]. However, it was never applied to adhesive contact of rough surfaces. In the present paper we fill this gap and simulate adhesive contacts between randomly rough fractal surfaces with the method of reduction of dimensionality.

2. The model

According to the method of reduction of dimensionality, the contact between a rigid body with a rough surface and an elastic half-space can be substituted by a one-dimensional contact problem with an elastic foundation, as illustrated in Fig. 1. To be equivalent to the initial three-dimensional problem, the stiffness of each spring of the elastic foundation must be chosen according to the rule

\[ \Delta k = E^* \Delta x, \]

where \( \Delta x \) is the space between adjacent springs (discretization step) and

\[ E^* = \frac{E}{1 - \nu^2}, \]

\( E^* \) being the Young modulus of the elastic half-space and \( \nu \), its Poisson ratio.

Additionally, for adhesive contacts, the following rule of Hess [22, 25] must be applied. Consider a one-dimensional substitution system sketched in Fig. 1. If the upper body is first pressed onto underlying elastic foundation and then pulled off, then the most stressed outer springs will detach from the body when they achieve the critical length

\[ \Delta l_{\text{max}}(a) = \frac{2\pi \alpha \nu_{12}^3}{E}, \]

where \( 2a \) is the length of the contact. It has been shown in [22] that application of this rule provides exact results for adhesive contact with an arbitrary body of revolution with respect to relations of the normal force \( F_n \), the indentation depth \( d \) and the contact radius \( a \).

One further rule is needed in the case of randomly rough surfaces. A randomly rough surface \( z(x) \) can be characterized by its spectral density

\[ C_{2D}(q) = \frac{1}{(2\pi)^2} \int \langle z(x)z(0) \rangle e^{-iq \cdot x} d^2x, \]

where \( \langle \ldots \rangle \) means ensemble averaging. In the case of self-affine fractal surfaces, the spectral density has the form [4]

\[ C_{2D} = \text{const} \begin{pmatrix} q \overline{q} \\ q_0 \end{pmatrix}^{-2(H+1)}, \]

where \( H \) is the Hurst exponent. It is directly associated with the fractal dimension \( D_f \) of the surface: \( D_f = 3 - H \). In the three-dimensional physical space, \( D_f \) can change between 2 and 3 so that the Hurst exponent takes values in the range of \( 0 \leq H \leq 1 \). Typical values for real physical surfaces are around \( D_f = 2.3 \) and \( H = 0.7 \). Generally, the validity of the power law (5) is limited by some cut-off wave vectors \( q_{\text{min}} \) and \( q_{\text{max}} \). In the present paper, we consider surfaces without cut-off at the lower limit of the wave vectors (or large wave lengths). The only natural cut-off is due to the size \( L \) of the system:

\[ q_{\text{min}} = \frac{\pi}{L}. \]

It was argued in [20] that the contact with a one-dimensional elastic foundation will be equivalent to the three-dimensional problem if the spectral density \( C_{1D}(q) \) of the one-dimensional equivalent “rough line” is defined as

\[ C_{1D}(q) = \pi q C_{2D}(q). \]

The one-dimensional profile is generated according to the rule

\[ h(x) = \int_{q_{\text{max}}}^{q_{\text{min}}} dq B_{1D}(q) \cos(qx + \varphi(q)), \]

where \( \varphi(q) \) is a random phase and

\[ B_{1D}(q) = \frac{2\pi}{L} C_{1D}(q) = B_{1D}(-q). \]

In Fig. 2, typical profiles of equivalent one-dimensional rigid lines for two values of the Hurst exponent are shown. Furthermore, the current position of the elastic (upper) sur-

![Fig. 1. Schematic representation of the one-dimensional model used as well as of the “rule of Hess” for the adhesion criterion](image-url)