Fluorescence Spectrum of a One-Atom Laser in the Strong-Coupling Regime

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Abstract—The multicomponent spectrum of the resonance fluorescence is studied based on the expansion of the Green function with respect to the coherent states for a one-atom laser. An analytical solution is derived for the spectrum in the frequency range corresponding to the vacuum Rabi oscillations in the approximation of the strong atom–field coupling and a low incoherent pump intensity.

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1. INTRODUCTION

The experimental realization of a one-atom laser in the regime of the relatively strong coupling of an atom and the cavity field mode has made it possible to observe several nonclassical effects in the laser radiation, including the photon antibunching and the sub-Poissonian statistics of light [1–4]. In addition to the measurement of the radiation of the cavity field mode, one can study the atomic emission to the noncavity field modes (atomic resonance fluorescence (RF)). The theoretical analysis of the RF spectrum of a one-atom laser (maser) can be found in [5] and the references therein. The distinctive features of these spectra (multipeak character and the formation of ultranarrow resonances whose width is less than the natural line width) have been analyzed based on the numerical calculations on the basis of the Fock states.

In this work, we consider the RF spectrum of a one-atom laser based on a method involving the expansion of the Green function of the quantum-system evolution with respect to the coherent field states. This approach makes it possible to derive analytical expressions for the RF spectrum and the explicit dependences of the widths and positions of the resonance lines on the parameters of the system in the approximation of the strong coupling and a low incoherent pump intensity.

2. MODEL

The simplest model of a one-atom laser represents an incoherently excited two-level atom with the ground |1⟩ and excited |2⟩ states that is placed in a high-Q optical cavity. In various regimes, this system can serve as a source of radiation whose statistics is changed from super- to sub-Poissonian. The working regime of a one-atom laser is determined by the relation between four parameters: (i) constant $g$ of the atomic coupling with the cavity field mode for the Jaynes–Cummings Hamiltonian $H_{\text{int}} = ig(a^*\sigma_z - a\sigma_x)$, where atomic ($\sigma_z, \sigma_x$) and field ($a, a^*$) operators satisfy the commutation relationships $[\sigma_z, \sigma_\pm] = \sigma_\pm$ and $[a, a^*] = 1$ for the Fermi and Bose particles, respectively; (ii) decay rate $k$ of the cavity field mode; (iii) rate of decay $R_{12}$ of the excited atomic state |2⟩ to the ground state |1⟩; and (iv) rate $R_{12}$ of the incoherent atomic pumping at the $|1⟩ \rightarrow |2⟩$ transitions.

Variations in the working regimes of a one-atom laser are accompanied by variations in the spectrum of the atomic emission to the noncavity field modes (the RF spectrum). The RF spectrum can be calculated using the solution to the control equation for the density matrix reduced with respect to the states of the environment

$$\dot{\rho} = L\rho, \quad (1)$$

where the Liouvillian $L = 1/i\hbar[H_{\text{int}}, \rho] + L_\text{of} + L_\text{sp}$ consists of the term that describes the atomic coupling with the resonance field mode ($1/i\hbar[H_{\text{int}}, \rho]$), the Liouvillian that describes the incoherent atomic pumping and atomic relaxation ($2L_\text{of} = R_{12}[(\sigma_z\rho, \sigma_\pm) + [\sigma_z, \rho\sigma_\pm]] + R_{21}[(\sigma_\pm\rho, \sigma_z) + [\sigma_\pm, \rho\sigma_z]]$), and the Liouvillian that describes the field decay in the cavity ($L_\text{sp} = k([a\rho, a^*] + [a, \rho a^*])$).

3. GREEN FUNCTION REPRESENTED IN TERMS OF THE COHERENT FIELD STATES

To determine the RF spectrum and the remaining spectral and temporal characteristics of the system under study, we need to know the nonstationary solutions to Eq. (1) at various initial conditions. The struc-
ture of this equation prompts the application of the rep-

\[ \rho(t) = \sum_{i,j=1,2} |i\rangle \langle j| \int d^{2}\alpha P_{ij}(\alpha, t) |\alpha\rangle \langle \alpha|, \]

where the functions \( P_{ij}(\alpha, t) \) are the distribution functions for the corresponding elements of the atomic density matrix: \( \rho_{ii} = S_{\alpha} \rho: \langle i | \rho_{ii}(t) | j \rangle = \int d^{2}\alpha P_{ij}(\alpha, t) \) To find two-time correlation functions, for example, the atomic function \( \langle \sigma_{+}(t + \tau) | \sigma_{-}(t) \rangle \) that determines the RF spectrum [6, 7]

\[ \langle \sigma_{+}(t + \tau) | \sigma_{-}(t) \rangle = \sum_{i} \int d^{2}\beta \int d^{2}\alpha P_{i21}^{kl}(\beta, t + \tau | \alpha, t) P_{21}(\alpha, t), \]

we need to know the transient conditional distribution functions represented in terms of coherent states \( P_{ij}^{kl}(\beta, t + \tau | \alpha, t) \) for the element \( \langle i | \rho_{kl}(t + \tau) | j \rangle \) of the atomic density matrix under the condition that the system is found in coherent state \( |\alpha\rangle \) at moment \( t \) and the only nonzero element of the atomic density matrix is \( \langle k | \rho_{ii}(t) | l \rangle = 1 \). In other words, we need to know the functions \( P_{ij}^{kl}(\beta, t + \tau | \alpha, t) \), which represent the Green functions for Eq. (1) in the coherent-state representation:

\[ P_{ij}^{kl}(\beta, t | \alpha, t) = \delta_{ik} \delta_{jl} \delta(\beta - \alpha), \]

For the stationary lasing, correlation function (3) only depends on time difference \( \tau \). In this case, we choose point \( t = 0 \) as the starting point and represent the normalized stationary RF spectrum as a Fourier transform of correlation function (3):

\[ S_{\sigma}(\omega) = 2 \text{Re} \int d\tau \exp(i \omega \tau) \langle \sigma_{+}(\tau) | \sigma_{-}(0) \rangle / \langle \sigma_{+}(0) | \sigma_{-}(0) \rangle, \]

where \( \langle \sigma_{+}(0) | \sigma_{-}(0) \rangle \) is the stationary population of the excited atomic state.

To find the stationary spectrum, we must know the solutions to both the stationary problem for the probability densities \( P_{ij}(\alpha, 0) \) and the nonstationary problem for the conditional probability densities \( P_{ij}^{kl}(\beta, \tau | \alpha, 0) \). Both probability densities satisfy the equation for the Glauber \( P \) function [8]. In the same work, one can also find an analytical stationary solution for the quasi-probability \( P \) function. In this work, we will find the nonstationary solutions for the conditional probability den-

\[ F_{ij}^{kl}(\lambda, \tau | \alpha, 0) = \int d^{2}\beta P_{ij}^{kl}(\beta, \tau | \alpha, 0) e^{\lambda^{*} \beta - \lambda \beta}, \]

which are equal to the matrix elements of the conditional characteristic field operator

\[ F_{ij}^{kl}(\lambda, \tau | \alpha, 0) = \delta_{ik} \delta_{jl} \exp(\lambda^{*} a) \exp(-\lambda a^{*}) \rho_{ij}^{kl}(\tau | \alpha, 0) \),

where \( \rho_{ij}^{kl}(\tau | \alpha, 0) \) is the superoperator (Green matrix) of the system under study, which satisfies Eq. (1) and the following initial conditions:

\[ \rho_{ij}^{kl}(\tau = 0 | \alpha, 0) = |k\rangle \langle l| \alpha \rangle \langle \alpha|, \]

In accordance with definition (7), the initial values of the conditional characteristic operator are given by

\[ F_{ij}^{kl}(\lambda, 0 | \alpha, 0) = \delta_{ik} \delta_{jl} e^{\lambda^{*} a - \lambda a}. \]

In accordance with formula (6), correlation function (3) is reduced to

\[ \langle \sigma_{+}(\tau) | \sigma_{-}(0) \rangle = \sum_{i} \int d^{2}\alpha F_{i21}^{kl}(\lambda = 0, \tau | \alpha, 0) P_{21}(\alpha, 0). \]

Thus, two elements of the conditional characteristic operator \( F_{i21}^{kl} \) and \( F_{21}^{kl} \) are needed to find the RF spectrum.

In accordance with Eq. (1), the evolution of the characteristic operator \( F(\lambda, t | \alpha, 0) \) is given by equation

\[ \partial F/\partial t = -g(\lambda^{*} \sigma_{+} F + g^{*} \sigma_{-} F) + g(\sigma_{+} \partial F/\partial \lambda^{*}) + g(\sigma_{-} \partial F/\partial \lambda) - L_{P} - k(\sigma_{+} \partial F/\partial \lambda^{*} + \lambda \partial F/\partial \lambda). \]

Using the basis of the atomic variables, one can represent Eq. (11) as an equation for the four-component vector \( \vec{F} = (\vec{F}_{1}, \vec{F}_{2}, \vec{F}_{3}, \vec{F}_{4})^{T} \):

\[ \vec{F}_{1} = \langle 1 | F | 1 \rangle + \langle 2 | F | 2 \rangle, \quad \vec{F}_{2} = \langle 1 | F | 2 \rangle - \langle 2 | F | 1 \rangle, \quad \vec{F}_{3} = \lambda \langle 1 | F | 2 \rangle, \quad \vec{F}_{4} = \lambda^{*} \langle 2 | F | 1 \rangle. \]

After the change of variable \( \lambda = re^{i\theta} \), this equation is represented as

\[ \frac{\partial \vec{F}}{\partial t} = M \left( r, \frac{\partial}{\partial r}, \frac{\partial}{\partial \theta} \right) \vec{F}, \]

where matrix \( M \) is written as