Vortex Dynamics in Bose–Einstein Condensates: Numerical Calculations

D. M. Jezek* and H. M. Cataldo

Consejo Nacional de Investigaciones Científicas y Técnicas and Departamento de Física,
Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires,
Ciudad Universitaria, (1428) Buenos Aires, Argentina

*e-mail: djezek@df.uba.ar
Received September 22, 2008

Abstract—We numerically study properties of the dynamics of vortices in nonrotating Bose–Einstein condensates in the Thomas–Fermi regime. On the one hand, we compute the vortex energy as a function of its position and we predict, using the expression of the Magnus force, the vortex precession velocity. On the other hand, we calculate the temporal evolution of the vortex-state and test the accuracy of the previous prediction. We also investigate the validity of analytical formulae of this velocity involving the healing length. In addition, we analyze the velocity field and the angular momentum and we compare them to available analytical expressions.

PACS numbers: 03.75.Lm, 03.75.Hh, 03.75.Kk
DOI: 10.1134/S1054660X09040094

1. TRAPPING POTENTIALS

The harmonic potential has the usual form

\[ V_h(r, z) = \frac{1}{2} m \omega_r^2 r^2 + \frac{1}{2} m \omega_z^2 z^2, \]

where \( \omega_r \) and \( \omega_z \) denote the radial and axial frequencies, respectively, and \( m \) is the particle mass. And the polynomial potential reads [5]:

\[ V_p(r, z) = \frac{1}{2} m \omega_r^2 r^2 \left( \frac{r-r_1}{r_1 r_2} \right) + \frac{1}{2} m \omega_z^2 z^2, \]

where \( r_1 \) and \( r_2 \) denote numerical parameters allowing to model the shape of the potential. We have worked in the limit \( \omega_r \gg \omega_z \), and thus the wavefunction can be factorized in the \( r \) and \( z \) cylindrical coordinates as the product \( \Psi(r) = \Psi(r) f(z) \), being \( f(z) \) a Gaussian function [6]. In particular, we have fixed \( \omega_r = 2\pi \times 100 \text{ Hz} \) and \( \omega_z = 200\omega_r \). Our condensates are formed by Rubidium atoms with a number of particles \( N = 10^5 \). The function \( \Psi(r) \) is obtained by solving the 2D Gross–Pitaevskii (GP) equation:

\[ \left( -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{trap}}(r, 0) + g_{2D} |\Psi(r)|^2 \right) \Psi = \mu \Psi(r), \]

where the effective two-dimensional coupling constant between the atoms is \( g_{2D} = 0.34 \) for the present system.

In Figs. 1a and 1b we show the reduced ground state densities \( \rho(r) = |\Psi(r)|^2 \) for both trapping potentials. The number of particles is large enough to assume that the Thomas–Fermi (TF) approximation is appropriate to describe the ground state density, which thus may be obtained by neglecting the kinetic term in the above expression. Therefore, the shape of this function has the form of the respective inverted potentials. On the other hand, the vortex states are numerically obtained by phase imprinting methods. The corresponding density profiles \( \rho_v(r) \) (Fig. 1) are similar to that of the ground state \( \rho_0(r) \), except for the presence of the vortex core.
2. VORTEX PRECESSION VELOCITY IN INHOMOGENEOUS MEDIA

When the energy $E$ depends on the vortex position $\mathbf{r}_0$, the vortex velocity $\mathbf{v}_p$ may be extracted from the following expression [7], related to the Magnus Force [8]:

$$2\pi\hbar\rho_0(\mathbf{r}_0)(\mathbf{z} \times \mathbf{v}_p) = \nabla E(\mathbf{r}_0),$$

being $\mathbf{r}_0 = |\mathbf{r}_0|$. For this purpose we have numerically computed the energy $E$ as a function of the vortex position for both trapping potentials. In Fig. 2 we depict the vortex energy $E_v = E - E_0$, being $E_0$ the ground state energy. We also plot in that figure a theoretical estimate of the vortex energy $E_v^{\text{theor}}$, which is displayed as solid lines. Such an estimate has been derived [9, 10] under the hypothesis that the size of the vortex core is of the order of the healing length $\xi$, which is utilized as a cutoff parameter in the corresponding integral. Thus, the theoretical prediction reads

$$E_v^{\text{theor}}(\mathbf{r}_0) = \frac{\pi\hbar^2}{m} \rho_0(\mathbf{r}_0) \ln(R_{TF}/\xi),$$

where $R_{TF}$ denotes the condensate radius and $\xi = \sqrt{\hbar^2/(2m g_{2D} \rho_0(\mathbf{r}_0))}$. Figure 3 shows the vortex pre-