A mathematical model of a growing cavern is used to describe possible scenarios of sinkholes in the karstic areas. The authors present formation criteria for ground surface sinkholes and underground caverns, and estimate their sizes.

Karst, rock mass, stress-strain state, collapse, cavern, sinkhole

INTRODUCTORY REMARKS

Karst is one of the most complex, hazardous and hardly predictable geological processes [1]. It spreads over 210 thousand square kilometers in Russia. Civil and industrial engineering on karsted terrain is unsafe. Although construction standards incorporate allowance for karstification into construction design [2, 3], they lack strict estimates of karst impact on buildings and structures, and are limited to probabilistic-statistical approaches and zoning according to karst intensity [3, 4]. Meanwhile it is obvious that the forecast of risk of karst and the estimate of its parameters must include both geological and engineering analysis and methods of the mechanics of continua and discrete media.

Many researchers tried to describe stress-strain state of rocks around a karst cavern. For example, Al’bov explained origin of sinkholes and karst subsidence using ideas and formulas by Protod’yakonov and Briggs. Collapse sinkholes were analyzed in the elastoplastic formulation [6], state of rocks with a cavern was assessed using the finite element method [7]. Subsidence and collapse of underground cavities [8, 9] were described by means of the discrete element method (DEM) [10]. The DEM-based approaches served in studying deformation [11] and stress-strain state [12] of rocks in the vicinity of caverns, as well as in analyzing karst evolution under technogeneous impact [13]. The authors [14] used a discrete deformation analysis, which is a branch of DEM, to relate ultimate span of roof of a cavern with the cavern occurrence depth in a laminated jointy rock mass.

The procedure of qualitative estimate of karst hazards over structures on land surface, based on mathematical modeling of alternated stress-strain state in the subsurface due to karst development was presented in the works [15, 16]. Parametrization of the geomechanical calculations was assisted with localization of caverns by 3D seismology methods and interpretation of the karst damages. The karst risk and hazards toward buildings and structures on the ground surface were evaluated from the maximum estimated / ultimate allowable strains (compressions and tensions) of the ground surface within the limits of a particular building or structure [17].

Zoning of karst terrains according to the karst hazard [18] was executed by estimating the maximum possible impact of karst on buildings and structures. The karst attack was supposed maximum when caverns reached the greatest possible size determined from geological conditions in
karsted rocks. Existent procedures described the single mechanism of collapses on the ground surface, i.e. full collapse of overlying rocks into a cavern [11]. Sinkholes in this case are dozens of meters in size. But, for instance, in the sulfate karst areas in Perm Region, sinkholes are first meters in size. Therefore, the present authors think, it is necessary to refine the sinkhole formation model and rock collapse mechanism.

A karst process can develop in the following stages: (1) formation of a cavity; (2) growth of the cavity under leaching; (3) reaching of ultimate dimension; (4) collapse of overlying rocks into the cavity (plastic and granular rocks), further growth of the cavity and the associated deformation of the earth surface; (5) gradual fill of the cavity with rocks and formation of a sinkhole on the earth surface.

1. GENERAL FORMULATION OF THE PROBLEM

The estimate of the karst hazards over building and structures does not need a comprehensive analysis of hydrochemical processes that result in formation of a cavern. It is sufficient to determine initial size and shape of the cavern, so that not to admit appreciable error into the problem solution.

According to the analysis of stress-strain state in rocks with a cavern, the rock stability is almost independent of the cavern height [17] and utterly depends on the cavern width (lateral dimension). Let initial width of a cavern, \(a_c\), be adopted from the cavern roof stability condition, and initial height of the cavern, \(h_c\), be evaluated using the data from [19], where examination of the Kungur ice cave (sulfate karst in Perm Region) established the lateral to vertical dimension ratio range in the caverns as 3 through 5.

It is assumable, without loss of generality, that a cavern forms at the bottom of karsted rocks, which supposes its potential to reach maximum dimension and allows estimation of upper limits of the karst impact on buildings and structures. Then, thickness of sulfate rocks, \(m_r\), and occurrence depth of karsted rocks, \(H_r\), will be determinative in evaluation of limit sizes of caverns and the rock collapse mechanism. Inclusive of the afore-mentioned, Figure 1 shows the calculation scheme for the initial state of a growing cavern in the sulfate karst in the standard geological profile for Perm Region.

Boundary conditions were set based on in situ stress state in the top rocks: horizontal displacements at the side boundaries and vertical displacements at the bottom boundary were assumed zero. The calculation domain was supposed to experience volume force \(\gamma_f\) (specific weight).

![Fig. 1. Standard calculation scheme.](image-url)