ROCK FAILURE

Growth of Hydrofractures in an Oil and Gas Stratum under Impulse Loading

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Abstract—It is analyzed how hydrofractures grow from the circular hole boundary in the conditions of plane strain, as well as how amount of hydrofractures, their initial length, working fluid pressure and external compression parameters affect the fractured zone form, opening of the cracks and their volume. Depending on the basic parameters, development conditions are determined for two or more cracks.

Keywords: system of created fractures, compression field, failure zone, crack opening

INTRODUCTION

The previous analyses of plane problems on equilibrium and quasi-static crack growth from boundary of a circular hole \([1-5]\) focused on influence exerted by biaxial compression field and free surface on the configuration of the crack growth path and the formation of failure area. These problems become of interest today in the context of actuality of optimizing the practice of flow rate enhancement when it is required to comprehensively analyze scenarios of propagation of many fractures from the well boundary. The multiple hydraulic fracturing quality parameters are fracture profiles and volumes. In order to estimate the change in the fracture profiles and volumes versus external stress field, paths of the fractures and their amount, the problem illustrated schematically in Fig. 1 is studied.

1. FORMULATION OF THE PROBLEM

The well axis \(Oz\) in the plane \(z = 0\) has a boundary represented by a circle with radius \(R\). At the boundary there are \(N\) incipient cracks with a length \(l_0(j)\) at inclined at the angles \(\alpha_j\) \((j = 1, N)\) to the boundary at the initial time. Natural compressive stresses \(\sigma_1\) and \(\sigma_2\) act at infinity. The maximum compression is oriented in parallel to the axis \(Ox\). Rock is modeled as an isotropic elastic body in plane strain conditions. The initial time pressure in the well is \(P(t)\). The working fluid is assumed perfect: it instantly fills growing cracks and only transfers normal stresses. Consequently, the boundary conditions at the well contour and on the crack surfaces are: \(\sigma_n = -P(t), \quad \tau_s = 0,\) where \(\sigma_n\) and \(\tau_s\) are normal and shear stresses, respectively.

\[\begin{array}{c}
\sigma_2 \\
y \\
\sigma_1 \\
\end{array}\]

Fig. 1. Schematic problem under analysis.
At any time, the crack contour \( L_k \) in its local coordinates has the form:

\[
t_k = \omega_k(\xi) = x_k(\xi) + iy_k(\xi), \quad |\xi| \leq 1, \quad k = 1, N, \quad i = \sqrt{-1}.
\]  

(1)

Using integral expressions of complex potentials \( \Phi(z) \) and \( \Psi(z) \) for the plane with a circular hole and cracks growing from its contour [6] reduces the problem to finding \( N \) unknown functions \( g_k' (\xi) \) in the system of \( N \) complex singular integral equations [1-4]. An additional condition to close the system of equations is the condition of finite displacements of the crack edges at the hole contour. In this case, \( g_k' (\xi) = g_k' (t_k) \cdot \omega_k(\xi) \), and \( g_k' (t_k) \) are proportional to derivatives of displacement discontinuities:

\[
\frac{d}{dt_k} \bigl\{ [U_k] + i[V_k] \bigr\} = \frac{4(1-v^2)i}{E} g_k' (t_k), \quad t_k \in L_k,
\]

(2)

where \([U_k], [V_k]\) are horizontal and vertical displacement discontinuities at the crack surfaces, respectively; \( E \) is Young’s modulus; \( v \) is Poisson’s ratio.

The system is solved in the form of:

\[
g_k' (\xi) = \frac{\varphi_k(\xi)}{\sqrt{1 - \xi^2}}.
\]

Gaussian formulas transform the system of integral equations into a system composed of \( 2Nn \) linear algebraic equations, where \( n \) defines the approximation order—the solution accuracy. The resultant system solution yields values of \( \varphi_k(\xi) \) \( (k = 1, N) \) at nodes \( \xi_j = \cos \frac{\pi(2j-1)}{2N}, \quad j = 1, n \), which are zeros of Chebyshev’s polynomial \( T_n(\xi) = \cos(n \arccos(\xi)) \).

Placing the found \( \varphi_k(\xi_j), \quad k = 1, N, \quad j = 1, n \) in the expressions of \( \Phi(z) \) and \( \Psi(z) \), by formulas by Kolosov-Muskhelishvili, allows finding stress state in the studied area. The profile of opening of a \( k \)-th crack—the normal displacement discontinuity lengthwise the crack—is calculated from:

\[
[V]_k(\xi_0) = \frac{-4(1-v^2)}{E} \sigma \frac{l_k}{R_k} \begin{matri

\[
\begin{pmatrix}
\alpha_k(\xi_0) \\
\alpha_k(\xi_0)
\end{pmatrix}
\begin{pmatrix}
\varphi_k(\xi) \\
\varphi_k(\xi)
\end{pmatrix}
\right] \frac{T_n(\xi_j)}{\sqrt{1 - \xi_j^2}} d\xi,
\]

where \( \xi_0 \) is a point at the crack contour; \( l_k \) is the half-length cord connecting the crack tips, and the interpolation polynomial by Lagrange for the function \( \varphi_k(\xi) \) has the form [6]:

\[
\varphi_k(\xi) = \frac{1}{n} \sum_{j=1}^{n} (-1)^{j+1} \varphi_k(\xi_j) \frac{T_n(\xi_j)}{\sqrt{1 - \xi_j^2}}.
\]

The crack growth paths in the quasi-static regime are calculated step-by-step, using the criterion “\( \sigma_{\theta \theta} \)” [7]. At each step, the problem on \( N \) curved cracks is solved, the profiles of the crack opening are found, and the following formula:

\[
K_{Lk} - iK_{\|k} = -\sigma_1 \sqrt{l_k} \cdot \sqrt{\left| \omega_k'(1) \right|} \cdot \frac{\varphi_k(1)}{\omega_k'(1)}
\]

is used to determine stress intensity factors at the right-hand tips of the cracks \( L_k \) (the left-hand tips are at the hole contour). The direction of the further propagation of the cracks is determined by the angle:

\[
\theta_k^* = 2\arctan \frac{K_{Lk} - \sqrt{K_{Lk}^2 + 8K_{\|k}^2}}{4K_{\|k}}, \quad K_{Lk} > 0.
\]