Constructive Methods for Synthesizing Inhomogeneous Layered Structures under the Effect of Elastic Waves

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Abstract—A variational statement of the problem of optimal synthesis aimed at obtaining inhomogeneous layered structures that exhibit a required set of properties under the effect of elastic waves is studied. The possibility of targeted control of mode conversion at the boundaries between elastic layers is investigated with a view to extending the limits in designing structurally inhomogeneous systems with preset properties. The optimum conditions are presented for problems of optimal synthesis of inhomogeneous layered structures under the effect of elastic waves in terms of the aforementioned variational statement. Analytical relations are derived for an effective a priori contraction of the allowable set of materials, which makes it possible to increase the efficiency of the search for optimal solutions and to extend the limits of applicability of different approaches.

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VARIATIONAL STATEMENT OF THE OPTIMAL SYNTHESIS PROBLEM

In connection with the wide application of composite materials, structures, and coatings with inhomogeneous layered structures in modern instrument making, it is of interest from both theoretical and applied points of view to study the possibilities for an efficient control of the energy characteristics of wave processes on the basis of a purposeful choice of the geometric and physical structures of composite systems [1, 2]. Studies of inhomogeneous layered structures and their interaction with wave processes are the subject of many recent publications [3–14].

The present paper considers the case where an inhomogeneous layered structure contains systems of elastic layers. The possibility of the excitation of two types of waves, namely, longitudinal and shear waves, in a system of elastic layers, as well as the possibility of mode conversion at the boundaries between the layers, leads to the formation of a more complicated interference pattern, as compared to the case where only one type of waves can propagate in a composite structure [3, 4, 6, 8]. A considerable complication of the interference phenomenon is primarily caused by the possibility of mutual conversion of two types of waves at the boundaries. The problem under consideration is related to studying the possibility of controlling the mode conversion at the boundaries between elastic layers with the goal of extending the limits in the design of structurally inhomogeneous systems with required sets of properties.

Let the required parameters of an elastic wave be preset at the output of a composite system; i.e., let the quality criterion be preset. For example, the quality criterion may consist in the requirement that the type of motion be conserved, or one type of motion be transformed into another type for a given spectral interval, or the energy of elastic waves be suppressed in a sufficiently wide spectral range, or, the incident wave be transformed into a surface wave. It is necessary to adjust a composite structure so as to make the characteristics of the wave process at the output of the structure as close as possible to the required characteristics. The problems under consideration belong to inverse problems of mathematical physics [15].

Let us consider a nonmonochromatic elastic wave obliquely incident on a multilayer system consisting of N plane-parallel layers. The outer surface of the structure coincides with the xy plane, and the plane of incidence of the elastic wave coincides with the xz plane. The subsequent consideration will be restricted to the case where the infinite half-spaces surrounding the system of layers are ideal fluids. In this case, in the half-spaces bounding the system of elastic layers, no shear waves will be present.

The propagation of an elastic wave in a system of elastic layers is described by the following equations of elastic motion:

\[ \mu_s \Delta u_s + (\lambda_s + 2\mu_s) \text{grad} \text{div}(u_s) = \rho_s \frac{\partial^2 u_s}{\partial t^2}, \]

\[ (s = 1, \ldots, N). \]
Here, \( \mathbf{u}_s(x, y, z, t) \) is the particle displacement vector in the medium of number \( s, \rho_s \) is the density of the \( s \)th layer, and \( \lambda_s, \mu_s \) are the Lamé constants of the \( s \)th layer. The vector displacement field can be represented in the form of a superposition of two fields [16]:

\[
\mathbf{u}_s = \nabla \Phi_s + \text{curl} \mathbf{P}_s, \tag{2}
\]

where \( \Phi_s \) and \( \mathbf{P}_s \) are the scalar and vector potentials of the wave field. A plane wave of a general form can be represented as a superposition of plane harmonic waves; i.e., it can be represented in the form of the Fourier integral

\[
\Phi_s(x, z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f_s(z, \omega) \exp(i\Delta_0 x - i\omega t) d\omega, \tag{3}
\]

\[
P_s(x, z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g_s(z, \omega) \exp(i\Delta_0 x - i\omega t) d\omega,
\]

\((s = 1, \ldots, N).\)

Here, \( \Delta_0 = k_0 \sin \vartheta_0 \), \( k_0 = \omega/c_0 \) is the wave number of the incident wave, \( c_0 \) is the velocity of wave propagation in the first half-space, \( \vartheta_0 \) is the angle of incidence of the longitudinal wave on the inhomogeneous layered structure, and the functions \( f_s(z, \omega) \) and \( g_s(z, \omega) \) have the meaning of complex amplitudes of the scalar and vector potentials in the \( s \)th medium; these functions satisfy the following system of equations:

\[
\frac{\partial^2 f_s(z, \omega)}{\partial z^2} + (k_0^2 - \Delta_0^2(\omega)) f_s(z, \omega) = 0, \tag{4}
\]

\[
\frac{\partial^2 g_s(z, \omega)}{\partial z^2} + (\gamma_s^2 - \Delta_0^2(\omega)) g_s(z, \omega) = 0,
\]

\[
b_{s-1} \leq z \leq b_s, \quad s = 1, \ldots, N, \]

\[
f_s(b_{s-1}, \omega) = \Phi^s_l f_s(b_{s-1}, \omega) + i\Phi^s_2 \frac{\partial g_{s-1}(b_{s-1}, \omega)}{\partial z},
\]

\[
g_s(b_{s-1}, \omega) = \Phi^s_3 g_{s-1}(b_{s-1}, \omega) + i\Phi^s_4 \frac{\partial f_{s-1}(b_{s-1}, \omega)}{\partial z},
\]

\[
\frac{\partial f_s(b_{s-1}, \omega)}{\partial z} = Q^s_1 \frac{\partial f_{s-1}(b_{s-1}, \omega)}{\partial z} + iQ^s_2 g_{s-1}(b_{s-1}, \omega),
\]

\[
\frac{\partial g_s(b_{s-1}, \omega)}{\partial z} = Q^s_3 \frac{\partial g_{s-1}(b_{s-1}, \omega)}{\partial z} + iQ^s_4 f_{s-1}(b_{s-1}, \omega), \tag{5}
\]

The quantities \( L_i, M_i (i = 1, 2, 3) \) and \( C_i, D_i (i = 1, 2) \) depend on the physical characteristics of the contacting media at \( z = 0 \) and \( z = 1 \). The interfacing conditions in the boundary-value problem (4) are a consequence of the continuity of the normal and tangential stress and displacement components at the boundaries between layers with different physical properties.

The optimization criterion in the variational statement of the problem under study is taken to be the closeness of the energy transmission factor of the elastic wave \( T(\omega) \) to the required dependence \( \tilde{T}(\omega) \) in a given frequency range bounded by \( \omega_{\min} \) from below and \( \omega_{\max} \) from above:

\[
J = \int_{\omega_{\min}}^{\omega_{\max}} [T(\omega) - \tilde{T}(\omega)]^2 d\omega \Rightarrow \min. \tag{6}
\]

Here,

\[
T(\omega) = \frac{\Pi^r}{\Pi^{inc}} = \frac{c_0 \rho_0 \cos \vartheta_{N+1}}{c_{N+1} \rho_{N+1} \cos \vartheta_0} \text{mod}^2(f_{N+1}(l, \omega)),
\]

\[\text{mod}^2(f_{N+1}(l, \omega)) = \frac{\partial g_1(0, \omega)}{\partial z} = iM_1(\omega) + iM_2(\omega)f_1(0, \omega)
\]

\[+ iM_3(\omega)g_1(0, \omega),
\]

\[\frac{\partial f_N(l, \omega)}{\partial z} = iC_1(\omega) f_N(l, \omega) + iC_2(\omega) g_N(l, \omega),
\]

\[\frac{\partial g_N(l, \omega)}{\partial z} = iD_1(\omega) f_N(l, \omega) + iD_2(\omega) g_N(l, \omega).
\]