INTRODUCTION

Diffraction phenomena on strongly elongated bodies have their own specificities. In the case of bodies with a large wave size, numerical calculations are difficult (for a discussion of parameter values for which computations are possible, see [1]); in contrast, the lower boundary of the applicability domain of standard high-frequency asymptotics shifts to higher frequencies with an increase in the elongation rate of the body, and description of the wave field with the usual asymptotic formulas may be impossible for some problems. We develop a new shortwave asymptotic approach, which takes into account the fact that the body is strongly elongated and leads to new approximate formulas that in some sense possess the property of uniformity with respect to the parameter characterizing its elongation rate. The near field, that is, the field in a thin boundary layer near the surface of the elongated body, was studied in [2, 3].

According to the Kirchhoff principle, this near field can be considered as the source of the scattered field far from the surface. This paper is devoted to calculating the scattering characteristics of a strongly elongated body.

Note that the developed approach differs from the one used in [4] by the assumption that the spheroid is strongly elongated, but compared to [5] its transverse size is asymptotically large.

PREVIOUS RESULTS

The problem of diffraction of a plane wave incident at a small angle to the axis of a strongly elongated spheroid has been investigated in [3]. The strongly elongated spheroid is characterized by the following assumption on the asymptotic order of the parameters:

\[ \frac{\alpha^2}{b} = O(1) \]

where \( k \) is the wavenumber, and \( a \) and \( b \) are the small and the large semiaxes of the spheroid. In the boundary layer near the surface of the spheroid, stretched coordinates \((\eta, \tau)\) are introduced, which are related to cylindrical coordinates \((r, z)\) by the formulas:

\[ r = \frac{a^2}{b} \sqrt{1 - \eta^2} \sqrt{\tau}, \quad z = b\eta + \frac{1}{2} \frac{a^2}{b} \eta \tau. \]

The plane wave is incident on the spheroid at an angle \( \beta \) to the axis. It is assumed that the angle is small, so that

\[ \beta = \eta \sqrt{\frac{ab}{\tau}} = O(1). \]

The representation for the field in the boundary layer is constructed in the leading order by \( \sqrt{ab} \) according to the parabolic equation method. It has the following form:

\[
\begin{align*}
    u &= e^{ikb\eta} e^{-i\alpha \eta^2} \sum_{n=0}^{\infty} \sum_{\gamma=0}^{\infty} 2^\gamma \cos(\gamma \beta) \sum_{\delta=0}^{\infty} \frac{1}{1 + \delta} \int_{-\infty}^{+\infty} \left( \frac{1 - \eta}{1 + \eta} \right)^\mu \, d\gamma 
    \times M_{\mu,\gamma/2}(i\gamma \beta) \Omega_n(t) \left( M_{\mu,\gamma/2}(i\gamma \tau) + R_\alpha(t) W_{\mu,\gamma/2}(i\gamma \tau) \right) dt.
\end{align*}
\]

Here \( \delta_n \) is Kronecker symbol, \( M_{\mu,n/2} \) and \( W_{\mu,n/2} \) are Whittaker functions,

\[
\Omega_n(t) = \frac{1}{\pi} \frac{\Gamma(n/2 + 1/2 + it) \Gamma(n/2 + 1/2 - it)}{\Gamma^2(n + 1)},
\]

and functions \( R_\alpha(t) \) have the meaning of reflection coefficients. If one lets all \( R_\alpha = 0 \), formula (1) will give the incident wave, in the case of Dirichlet or Neumann boundary conditions on the surface of the
spheroid on should set \( R_n = R_n^{(S)} \) or \( R_n = R_n^{(H)} \), correspondingly, where
\[
R_n^{(S)}(t) = -\frac{M_{it, nt/2}(-i\chi)}{W_{it, nt/2}(-i\chi)},
\]
\[
R_n^{(H)}(t) = -\frac{M_{it, nt/2}(-i\chi) + 2i\chi M_{it, nt/2}(-i\chi)}{W_{it, nt/2}(-i\chi) + 2i\chi W_{it, nt/2}(-i\chi)}.
\]

In the case of axisymmetric incidence, \( \alpha = 0 \) and the ambiguity in formula (1) should be removed. In this case, only the term with \( n = 0 \) remains, and
\[
M_{it, 0}(\beta^2)\beta^{-1} \to e^{i\alpha/4}.
\]

Letting \( \tau = 1 \) in (1), it is easy to obtain the expressions for the field on the surface of the spheroid. The formulas for \( \partial u/\partial \tau \) on the surface of an acoustically soft spheroid and for \( u \) on the surface of an acoustically hard spheroid are presented in [3].

**SCATTERED FIELD**

Asymptotics (1) allows the far field in the problem of scattering by a strongly elongated spheroid to be found by means of the Kirchhoff integral [6]. Let \( G(\mathbf{r}, \mathbf{r}_0) \) be the Green’s function in the free space,
\[
G(\mathbf{r}, \mathbf{r}_0) = e^{ikr/2}\begin{vmatrix} \mathbf{r} - \mathbf{r}_0 \end{vmatrix},
\]
and \( U(\mathbf{r}) \) be the scattered field. Then
\[
U(\mathbf{r}_0) = \frac{1}{4\pi} \int_S G(\mathbf{r}(s), \mathbf{r}_0) \frac{\partial U}{\partial n} (\mathbf{r}(s)) \; ds,
\]
where \( S \) is the surface of the spheroid and \( n \) is the normal to the surface. The total field \( u \) can be put under the integration sign in (2) since the incident field does not contribute to the integral.

Following [6], let us introduce the amplitude of the field in the far zone \( \Psi \) using the asymptotics
\[
U = \Psi e^{ikr/\lambda}, \quad R \to +\infty,
\]
Then, as the point \( \mathbf{r}_0 \) tends to infinity and passing to the limit under the Kirchhoff integral yields
\[
\Psi(\vartheta, \vartheta_0, \varphi_0) = \frac{1}{4\pi} \int_S \left( \Psi_G(\mathbf{r}(s), \vartheta_0, \varphi_0) \frac{\partial U}{\partial n} (\mathbf{r}(s)) - \frac{\partial \Psi_G}{\partial n} (\mathbf{r}(s), \vartheta_0, \varphi_0) U (\mathbf{r}(s)) \right) \; ds, \tag{3}
\]
where angles \( \vartheta_0 \) and \( \varphi_0 \) specify the direction along which point \( \mathbf{r}_0 \) tends to infinity and \( \Psi_G \) is the amplitude of the point source in the far zone.

We shall consider small angles \( \vartheta_0 \), such that \( \vartheta_0 \sqrt{kb} = \beta_0 = O(1) \). Due to the reciprocity principle, \( \Psi_G \) coincides with the field of the plane wave incident at angle \( \beta_0 \) and has the asymptotics expressed by formula (1), in which one should change the sign of \( \eta \) and set \( R_n = 0 \). In other words,
\[
\Psi_G = e^{-ik\vartheta_0} \cos(n\varphi_0) \sum_{m=0}^{\infty} 2^m \frac{\cos\left[ m(\varphi - \varphi_0) \right]}{1 + \delta_0^m} \int_{-\infty}^{+\infty} \left[ M_{it, m/2}(\beta^2) M_{i\eta, n/2}(\beta^2) \right] \Gamma \left( \frac{\eta + 1}{2} + it \right) \Gamma \left( \frac{\eta + 1}{2} - it \right) \; dt.
\]

It is easy to express the element of the surface \( ds \) and the normal derivative in integral (3) in the coordinates of the boundary layer:
\[
ds = a\sqrt{1 - \eta^2} \; d\varphi d\eta,
\]
\[
\frac{\partial}{\partial n} \bigg|_{n=0} = \frac{1}{a\sqrt{1 - \eta^2}} \frac{\partial}{\partial \eta}.
\]

Let us substitute asymptotics (1) and (4) into integral (3) and change the order of integration. In view of the orthogonality of trigonometric functions, only the terms in which indices \( n \) and \( m \) coincide, make a non-zero contribution. Thus, a double series is reduced to a delta-function:
\[
\int_{a}^{b} \left[ 1 - \eta \right]^{1-i-s} \frac{d\eta}{1 + \eta} = \pi \delta(t - s),
\]
to see this, replace \( \eta = -\tanh(\lambda/2) \).

As a result, we find for the amplitude of the far field
\[
\Psi = \frac{2ib}{\pi \beta \beta_0} \sum_{n=0}^{\infty} (-1)^n \cos(n\varphi_0) \int_{-\infty}^{+\infty} M_{it, n/2}(\beta^2) M_{i\eta, n/2}(\beta^2) \Gamma \left( \frac{n + 1}{2} + it \right) \Gamma \left( \frac{n + 1}{2} - it \right) \; dt.
\]

The asymptotics of the scattering cross section [6]
\[
\Sigma = \frac{4\pi}{ka^2} \Im \Psi(\vartheta, \vartheta, \pi),
\]
which we normalize by the cross section of the spheroid \( 4\pi a^2 \), immediately follows from formula (5):
\[
\Sigma = \frac{8}{\pi \chi \beta^2} \sum_{n=0}^{\infty} \frac{1}{(n!)^2} \frac{1}{1 + \delta_0^n} \int_{-\infty}^{+\infty} \left[ 1 - \eta \right]^{1-i-s} \frac{d\eta}{1 + \eta} \left[ M_{it, m/2}(\beta^2) R_n(t) \Gamma \left( \frac{n + 1}{2} + it \right) \Gamma \left( \frac{n + 1}{2} - it \right) \right] \; dt. \tag{6}
\]

Let us rewrite formula (6) for numerical computations in terms of Coulomb wavefunctions \( F_L \) and \( H_L^+ \) (see [3]). For the case of an acoustically soft surface,