INTRODUCTION

The authors of [1] considered the one-dimensional inverse problem of finding, via the amplitude and phases, a Rayleigh wave recorded on the surface of a massif, as well as the velocities of longitudinal and transverse waves as a function of depth $z$. The similar problem in a two-dimensional formulation with a change in the Rayleigh wave velocity along the horizontal profile had not been considered previously.

In this connection, this article proposes a theoretical and experimental substantiation for applying the approaches developed in [1] to determine two-dimensional deep vertical seismic profiles with increasing steps of calculating transverse and longitudinal wave velocities using the theory of almost-periodic functions [2] and perturbation theory. The possibilities of the method are illustrated by the results of comparison with geological data obtained in regions of the Northern Caucasus using active seismics. We formulate the calculation stability conditions and present an example based on microseism data obtained by the Joint Institute for Physics of the Earth, Russian Academy of Sciences (OIFZ RAN) in an area of North Ossetia.

FORMULATION OF THE PROBLEM

Let us consider a stress-free, vertically and horizontally inhomogeneous elastic half-space $z \leq 0$ with a constant density and changing elasticity modulus depending on coordinates $x, z$. To solve the inverse problem of determining the seismic profile, functions of the coordinates and time of displacement of particles of a Rayleigh wave are given at points of the profile where this wave is detected. As well, there are no less than eight indicated points, and their step is half the wavelength at the predominant high frequency. In addition, it is assumed that the curvature of the profile (deviation from a straight line along the horizontal) is much less that the wavelength at the maximum of the analyzed frequencies, and the length of the profile is equal to the wavelength at the lowest frequency of the study.

A flat-layered medium is considered homogeneous in coordinate $y$. The propagation velocity of longitudinal and transverse waves is approximated by a function with a continuous second derivative; as well, horizontal changes in the velocities are much less than vertical changes.

The problem is solved either immediately for the entire profile relying on perturbation theory for the global phase velocity and the calculated local amplitudes and reference maximum and minimum velocities, or by current division into areas having their own phase velocities and amplitudes.

The solution to the inverse problem of determining the velocity of transverse (or longitudinal) waves is sought in the class of continuous functions expanded into a Taylor series and having a continuous second derivative.

SOLUTION OF THE PROBLEM

It was proved in [3] that division of variables leads, for Rayleigh waves, to the Sturm–Liouville problem, but in matrix form, for a number of coordinate systems, including Cartesian. Work [4] presents a system of equations, which arises after division of variables,
for plane monochromatic waves of vertical polarization.

In [1], it was shown via the Tricomi substitution that the equations for monochromatic waves transform to a form describing the interaction and transformation of the Rayleigh wave components:

\[
\frac{\partial^2 u}{\partial z^2} + \left[ \frac{\omega^2}{V_s^2(z)} - k^2 \lambda(z) + 2\mu(z) \right] u = \frac{\delta(z - \zeta)}{\mu(\zeta)} \ast f_1(z, \lambda, \mu),
\]

\[
\frac{\partial^2 w}{\partial z^2} + \left[ \frac{\omega^2}{V_p^2(z)} - k^2 \frac{\mu(z)}{\lambda(z) + 2\mu(z)} \right] w = \frac{\delta(z - \zeta)}{\mu(\zeta) + 2\mu(\zeta)} \ast f_2(z, \lambda, \mu),
\]

where \( \omega \) is the frequency; \( k \) is the wavenumber; \( \lambda(z), \mu(z) \) are the Lame constants; \( u, w \) are the Rayleigh wave particle displacement vector components; \( z \) is a coordinate; \( \zeta \) is the shift of the argument in convolution; and the \( * \) sign in the right-hand side of (1) denotes convolution,

\[
f_1(z, \lambda, \mu) = -ikk \frac{\lambda + \mu}{\sqrt{\lambda + 2\mu}} \times \left[ w' + w \left( -\frac{1}{2} \lambda + 2\mu + \mu' \right) \right],
\]

(2)

\[
f_2(z, \lambda, \mu) = -ikk \left[ \frac{\partial}{\partial z} (\lambda u) + \mu \frac{\partial u}{\partial z} \right].
\]

Convolution appears in connection with the interaction of \( P \) and \( SV \) waves and their transformation and leads to rotation of the Rayleigh wave components. Therefore, Eqs. (1) are considered as a system, and division of components occurs at the resonance frequencies of an \( SV \) wave for an orientation of the seismic receivers in the vertical plane. This ensures the following amplitude component relation: \( A_{SV} > A_P \). The resonance frequencies of a wave are isolated by sorting of the spectral amplitude in decreasing order. The requirement that the wavelength should be much larger than the size of the layer is necessary to represent the eigenfunction as the sum of the cosine and minimal integral addition. A small increment in the step for depth should lead to relatively small changes (increments) of the potential, the shear modulus, and the eigenfunction. This makes it possible to perform calculations at various steps independently of each other and avoid accumulation of errors when continuing at a fixed step. Thus, the depth of calculation is in fact limited only by the wavelength in a discrete spectrum.

In (1) for the \( SV \)-component of the Rayleigh wave, the eigenvalue is written as

\[
\lambda_{SV} = \frac{\omega^2}{V_s^2(0)} - k^2 \frac{\lambda(0) + 2\mu(0)}{\mu(0)},
\]

and for the \( P \)-component,

\[
\lambda_P = \frac{\omega^2}{V_p^2(0)} - k^2 \frac{\mu(0)}{\lambda(0) + 2\mu(0)}.
\]

Here \( K \) is the wavenumber of the Rayleigh wave. Analysis of expression (4) shows that the wavenumber for the \( SV \)-component is larger, and the phase velocity lower, than for the \( P \)-component.

For the \( SV \)-component in Cartesian coordinates \( X, Z \) (further we will understand \( X \) to mean a two-dimensional matrix of source and receiver coordinates), we find the direct Fourier transforms \( F_h(x, k, x_1, x_2) \) of signals at profile points \( x_1, x_2 \), and coordinate \( x \) (from the plus—minus of infinity up to a fixed \( x \)). According to V.S. Vladimirov [6], it is possible to obtain the Green’s function on a free surface \( z = 0 \) for the case of the dependence of the velocity of a transverse wave on the vertical coordinate \( z \):

\[
g(\omega, k, x_1, x_2) = F_1(\omega, k, x_1) F_1(\omega, k, x_2) / \mu(x_0) W(x_0),
\]

(6)

\[
x_1 \leq x_0 \leq x_2,
\]

\[
g(\omega, k, x_2, x_1) = F_2(\omega, k, x_1) F_1(\omega, k, x_2) / \mu(x_0) W(x_0),
\]

\[
x_2 \leq x_0 \leq x_1.
\]

Here, \( F \) is the direct Fourier transform of normalized—over the maximum of the amplitude—recordings in the base \( x_1 \) and current \( x_2 \) profile points, and \( F_{1,2} \) is the complex conjugate function to \( F_{1,2}(\omega, k, x_0) \). The Wronskian is equal to

\[
W(x_0) = F_0(\omega, k, x_0) (-ik) F_0(\omega, k, x_0)
\]

\[
- F_0(\omega, k, x_0) (ik) F_0(\omega, k, x_0)
\]

\[
= 2 F_0(\omega, k, x_0) (-ik) F_0(\omega, k, x_0).
\]

It is then obvious that the spectral matrix function is

\[
g(x_1, x_2, \omega, k) = \frac{F_1(\omega, k, x_1)}{2ik F_0(\omega, k, x_0) \mu(x_0)}, \quad x_0 = x_2,
\]

\[
g(x_2, x_1, \omega, k) = \frac{F_2(\omega, k, x_1)}{2ik F_0(\omega, k, x_0) \mu(x_0)}, \quad x_0 = x_1.
\]

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