A New Numerical Method for Solving the Acoustic Radiation Problem

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Abstract—A numerical method of solving the problem of acoustic wave radiation in the presence of a rigid scatterer is described. It combines the finite element method and the boundary algebraic equation one. In the proposed method, the exterior domain around the scatterer is discretized, so that there appear an infinite domain with regular discretization and a relatively small layer with irregular mesh. For the infinite regular mesh, the boundary algebraic equation method is used with spurious resonance suppression according to Burton and Miller. In the thin layer with irregular mesh, the finite element method is used. The proposed method is characterized by simple implementation, fair accuracy, and absence of spurious resonances.

Keywords: finite element method, boundary element method, boundary algebraic equations, discrete Green’s function

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INTRODUCTION

To solve numerically an outer diffraction problem, it is necessary to choose one of the numerous existing methods. The methods of solving such problems are divided into two classes: with discretization of space (e.g., [1, 2]) and with discretization of the boundary. The first class includes, for example, the finite difference method and the finite element method (FEM). The second class includes the boundary element method, methods of the theory of potentials, and discrete source methods.

The choice between the two classes is not easy, because each of them has its own advantages and disadvantages. Methods with discretization of space usually operate with great numbers of nodes, possess dispersion, and require special techniques for modeling radiation conditions. Methods with boundary discretization operate with full (not sparse) matrices, require fairly accurate calculation of singular integrals, and, in some cases, possess spurious resonances, i.e., numerical artifacts hampering the solution.

However, there exists one more class of methods that combines the advantages of the two aforementioned classes. It includes the boundary algebraic equation (BAE) methods [3–6]. In this class of methods, space is assumed to be initially discrete and a discrete analog of the boundary integral equation, i.e., a linear algebraic equation, is constructed. Previously, we studied the problem of spurious resonance suppression for BAE (see [7], where a comprehensive review of publications devoted to BAE and related subjects is presented). A disadvantage of BAE methods consists in that space is assumed to be discretized regularly; i.e., a periodic mesh is considered. This means that the shape of a scatterer is approximated by a set of mesh elements, such as identical squares, cubes, triangles, etc. In most cases, this leads to a considerable unjustified loss in accuracy. In this paper, we overcome the aforementioned drawback by combining the BAE and finite element methods. Namely, a layer lying between the regular boundary for which the boundary integral equation is formulated and the surface of an irregularly shaped scatterer is modeled by the finite element method.

PROBLEM STATEMENT

We consider a two-dimensional stationary radiation problem. Let smooth contour \( \Gamma \) with outer normal \( n \) lie in the \((x, y)\) plane (Fig. 1). In the external domain, Helmholtz equation is satisfied:

\[
\Delta u + k^2 u = 0. \quad (1)
\]
At the boundary, the field satisfies the Neumann condition

$$\frac{\partial u}{\partial n} = h,$$

(2)

where $h$ characterizes the sources. The field satisfies the radiation condition; i.e., for $k$ containing a small positive imaginary part, it decays at infinity. For a real $k$, it is possible to formulate the radiation condition in Sommerfeld form or to consider a limiting procedure with the imaginary part of $k$ tending to zero.

A two-dimensional problem is chosen to simplify the description and to make the figures more pictorial. Similar reasoning is suitable for a three-dimensional case.

The plane wave diffraction problem can easily be reduced to a radiation problem. For this purpose, the field is represented as the sum of the known incident wave and an unknown scattered wave. For the scattered wave, the radiation problem is formulated. In this case, the normal derivative of the scattered wave field at the boundary is identical to the normal derivative of the incident wave multiplied by $-1$.

DERIVATION OF BOUNDARY ALGEBRAIC EQUATIONS

We briefly describe the BAE method in the form proposed in [7]. Note that the terminology used by us in this paper slightly differs from that used in [7].

As an approximation to the problem described above, we consider the problem in a discretized space, i.e., on a regular mesh. An example of such a problem is shown on the left of Fig. 2. The mesh is assumed to be infinite. Some of the square elements of the mesh are removed. Contour $\Gamma'$ of the eliminated fragment represents the scatterer. We also consider an auxiliary mesh (at the right of Fig. 2), which is periodic, has the same geometric parameters, but has no eliminated elements.

At the nodes of meshes, nodal values of the field variable are preset. We assume that, by interpolation, it is possible, if necessary, to approximate the initial problem not only at nodal points. The nodes of the mesh are numbered by subscripts.

We assume that, by conventional methods, e.g., FEM, we can approximate Eq. (1) on the auxiliary mesh (at the right of Fig. 2) by the discrete equation

$$\sum_{n} \beta_{m,n} \bar{\pi}_{n} = 0 \text{ for all subscripts } m.$$  

(3)

Here, $\pi_{n}$ are nodal values of the field variable on the mesh at the right and $\beta_{m,n}$ are coefficients. The overbar indicates the quantities belonging to the auxiliary...