The Motion of a Star in the Vicinity of a Globular Cluster in an Elliptical Galaxy

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Abstract—The spatial motion of a star in the vicinity of a globular cluster located in an inhomogeneous, rotating elliptical galaxy (EG) is considered. Perturbations due to the gravitation of the galaxy are taken into account, taking it to be a two-layer system together with its halo: an inner ellipsoid, representing the luminous part of the galaxy, and a homeoid, representing space filled with dark matter between inner and outer ellipsoidal boundaries. The ellipsoids are taken to be homothetic and to have a common center, with the boundary of the outer ellipsoid coincident with the boundary of the galactic halo. The luminous part of the EG and the homeoid have different densities. The motion of the star near a globular cluster occurs outside the luminous part of the EG, but inside the homeoid. The concept of the “vicinity of the globular cluster” is concretized using the concept of a “sphere of influence” (and the gravitational sphere and Hill gravitational sphere). Stellar motions inside and outside the sphere of influence of the globular cluster are considered, and the region of possible motions is determined. A quasi-integral and surfaces of minimum energy are found, which under certain conditions can be transformed into an analog of the Jacobi integral and surfaces of zero velocity. The Lyapunov stability of the stationary solutions obtained is established. The results are applied to model EGs whose parameters coincide with those of NGC 4472 (M49), NGC 4636, and NGC 4374, which contain a large number of globular clusters, and are presented in the form of figures and tables. Using these galaxies as examples, it is shown that studying stellar motions, and also determining the libration points and establishing their stability, requires use of an exact, rather than an approximate, expression for the potential of the luminous part of the elliptical galaxy.

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1. FORMULATION OF THE PROBLEM

Stationary solutions for the motion of a passively gravitating globular cluster (GC) inside an inhomogeneous, rotating elliptical galaxy (EG) were considered in [1]. In that work, an EG with its halo was represented as a two-layer body, with an inner, inhomogeneous ellipsoid representing the luminous part of the galaxy and an outer region of space that is uniformly filled with dark matter, extending between inner and outer ellipsoidal boundaries. These bounding ellipsoids were taken to be homothetic and concentric; the space between them is called a homeoid. The outer boundary of the homeoid coincides with the boundary of the galaxy halo. The densities of the luminous part of the EG \( \rho^* \) and of the homeoid \( \rho_G \) were taken to be different. The motion of the GC occurred outside the gravitating luminous part of the EG, but inside the homeoid, treated like a perturbing body. Stationary solutions (libration points) were found for the GC, and their Lyapunov stability determined. It was shown using model elliptical galaxies MEG-1, MEG-2, and MEG-3 (analogs of NGC 4472, NGC 4636, and NGC 4374, which contain a large number of GCs), that identifying libration points and analyzing their stability requires the use of an exact, rather than an approximate, expression for the potential of the luminous part of the EG.

Here, we consider the spatial motion of a star in the vicinity of a GC that belongs to an EG. The motion of the star is studied both relative to the center of mass of the GC and in galactocentric coordinates, taking into account the gravitation of the inhomogeneous, rotating EG. As in [1], the EG with its halo is treated like a two-layer body. Its luminous part, or inner ellipsoid, has a density \( \rho^* = \text{const} \). The second layer is formed of the homeoid corresponding to the space between inner and outer homothetic, concentric ellipsoids, which is filled with dark matter with density \( \rho_G = \text{const} \). The outer boundary of the homeoid coincides with the boundary of the galactic halo.

Of course, a star moving in the vicinity of a GC will feel a gravitational attraction toward the cluster center. If the star is located sufficiently far from the GC, this force will be negligible. Therefore, to
more precisely define the concept of “in the vicinity,”
we consider the sphere of influence of the GClow.
Moreover, when analyzing the motion of a star in
the vicinity of a GC in galactocentric coordinates, we
must define the galactocentric center of mass of the
cluster.

Let $OXYZ$ be a set of coordinates with its ori-
ingen at the center of the EG, which is rotating with
constant angular velocity $\Omega$ about the polar axis $OZ$,
with the axes oriented along the axes of the EG. The
Cartesian coordinates $X, Y, Z$ of the center of mass
of the GC in these coordinates are determined by the
system of equations [1]

$$
\frac{d^2 X}{dt^2} - 2\Omega \frac{dY}{dt} = \frac{\partial U}{\partial X},
$$
$$
\frac{d^2 Y}{dt^2} + 2\Omega \frac{dX}{dt} = \frac{\partial U}{\partial Y},
$$
$$
\frac{d^2 Z}{dt^2} = \frac{\partial U}{\partial Z},
$$

where the force function (total potential) $U$ is given by
a sum of three terms:

$$
U = \frac{\Omega^2}{2} (X^2 + Y^2) + U^* + U_G.
$$

We took the potential $U$ to have the form (2) based
on the following reasoning. Let us suppose that the
parent EG consists of a luminous part, corresponding
to an ellipsoidal body with a (stellar) mass $M^*$ and
density $\rho^*$, and a non-luminous (dark matter) part,
corresponding to a homoeid with mass $M_G$ and
density $\rho_G$. As an example, Fig. 1 shows the luminous
part of MEG-3 (an analog of NGC 4472 =M49) with
mean density $\rho^*$ (unshaded region) and the homoeid
with mean density $\rho_G$ (shaded region). The dashed
line denotes the boundary of the GC and the luminous
part of the EG.

The explicit form of the potentials of the luminous
part of the EG $U^*$ and of the homoeid $U_G$ will be
presented below. The conventional deVaucouleur [2]
boundary of the luminous part of the EG and the
upper boundary of the homoeid (the boundary of the
galactic halo) were taken from the family of homo-
thetic (co-axial, concentric) ellipsoidal surfaces
described by the equation

$$
\frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2} = k^2,
$$

where $k = 0$ corresponds to the center of the EG,
$k = 1$ to the ellipsoidal surface with semi-axes $a, b,
and c$ of the galactic halo, and $0 < k < 1$ to elли-
psoidal surfaces with semi-axes $ka, kb, \text{and} \, kc$, which
bound the luminous part of the EG. Moreover, we
assumed that the major, minor, and polar axes satisfy
the inequalities $a \geq b \geq c$. For any $k$, the second
eccentricities of this family $\lambda$ and $\mu$ are related to the
semi-axes $a, b, \text{and} \, c$ by the expressions

$$
a^2 = c^2(1 + \lambda^2), \quad b^2 = c^2(1 + \mu^2)
$$

$$
(\mu^2 \leq \lambda^2, \quad \lambda^2 < 1),
$$

where the first inequality in parantheses flows from the
condition $a \geq b \geq c$; when the second inequality
($\lambda^2 < 1$) is satisfied, the potential $U_G$ is a polynomial,
which will be discussed below.

The potential of the luminous part of the galaxy
$U^*$, appearing in Eq. (2), is equal to [1, 3, 4]

$$
U^* = A \left( U_0 - U_1 X^2 - U_2 Y^2 - U_3 Z^2 \right),
$$

$$
A = \pi abc k^3 G \rho^* > 0,
$$

where $G$ is the gravitational constant, $\rho^*$ the mean
density of the luminous part of the EG, and the posi-
tive coefficients $U_n$ ($n = 0, 1, 2, 3$) will be defined
below. The quantity $AU_0$ is the value of the potential
$U^*$ at the center of the EG.

Finally, the potential $U_G$ of the homoeid comprised
of dark matter (the non-luminous part of the EG) with
the mean density $\rho_G$ is given by

$$
U_G = B \left( U_0 - U_1 X^2 - U_2 Y^2 - U_3 Z^2 \right),
$$

$$
B = \pi abc (1 - k^2) G \rho_G > 0 \quad (0 \leq k \leq 1).
$$

2. COEFFICIENTS OF THE GRAVITATIONAL
POTENTIALS $U^*$ AND $U_G$

We used the following formulas from [3, 4] to
determine the coefficients $U_n$ and $U_0$ ($n = 0, 1, 2, 3$).
The gravitational potential $V$ of an ellipsoidal body
with density $\rho$ bounded by the surface (3) at an ex-
ternal point $P(X, Y, Z)$ can be written

$$
V = \pi G \rho \pi abc k^3 \left( V_0 - V_1 X^2 - V_2 Y^2 - V_3 Z^2 \right),
$$

$$
(a \geq b \geq c, \quad V_3 \geq V_2 \geq V_1 \geq 0).
$$

Note that the expression for $V$ exactly coincides
with the expression for $U^*$ from (5) when $\rho = \rho^*$. The triple
inequality for the coefficients $V_n = V_n(k, s) \quad (n = 0, 1, 2, 3)$
in parantheses can easily be proven using the
condition $a \geq b \geq c$ and writing these coefficients
as improper integrals from some positive value of $s$ to
$\infty$ [1]:

$$
V_0 = \int_s^{\infty} \frac{du}{\Delta(u)} \geq 0,
$$

$$
V_1 = \int_s^{\infty} \frac{du}{(k^2 a^2 + u) \Delta(u)} \geq 0,
$$

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