1. INTRODUCTION

In the experiments with $e^+e^-$ annihilation to hadrons, the important role is played by the so-called “returning to resonance” mechanism. It consists in the emission of a hard real photon by initial leptons [1].

The Born contribution and the one-loop correction are taken into account in the Dirac tensor (the cross-symmetry partner of the Compton tensor—a bilinear combination of the hard photon emission currents averaged over lepton spin states and summed over photon polarization states). Infrared divergences are parameterized by introducing the “photon mass $\lambda$.

In the final expression, it is removed in a usual way by adding the contribution from additional soft photon emission.

We do not consider photon emission by the final charged particles and the effects of charge-odd interference of virtual or real photon emission from leptons and hadrons. Therefore, the Dirac tensor obtained in this way is universal.

The paper is organized as follow. In Section 2, the relation of the Dirac tensor to the cross section of radiative annihilation of a lepton pair to hadrons is clarified. We give the Born-level expression for the Dirac tensor and derive the general form of the radiative correction to it using the symmetry relation. In Section 3, we obtain the contribution arising from the mass operator of the positron and vertex function in the case where a positron and a photon are on the mass shell. In Section 4, we consider the contribution from the vertex function to the case of off-shell electron and the box-type Feynman amplitude with an electron, positron, and one of the photons on the mass shell. In Section 5, we analyze the total result for the Dirac tensor, adding the emission of additional soft photon contributions, which provide the final result that is free from the infrared divergences. The limit case of an almost collinear hard photon emission is considered and some numerical estimates are given.

We give the hadronic tensor for several final states:

$$\gamma^0 \rightarrow \pi^+\pi^-, \quad \mu^+\mu^-, \quad \rho^+\rho^-.$$

In Appendices A and B, the details of the calculation are presented. In Appendix C, the contribution to the Dirac tensor in the case of the emission of two hard photons is given.

2. GENERAL ANALYSIS

The Born-level matrix element of hard-photon emission by initial leptons in the process of high-energy annihilation of $e^+e^-$ to hadrons via a single virtual photon intermediate state

$$e^+(p_+) + e^-(p_-) \rightarrow \gamma^0(q) + \gamma(p_i) \rightarrow \gamma(p_i) + h(q) \quad (1)$$

has the form (see Fig. 1)

$$M = \left(\frac{4\pi\alpha}{q^2}\right)^{3/2} V(p_+) O^{(B)}_{\rho} u(p_-) H_{\rho}(q),$$

where

$$O^{(B)}_{\rho} = \gamma_{\rho} \hat{p}_- \gamma_{\rho} \hat{p}_1 \gamma_{\rho} e + \gamma_{\rho} \hat{p}_+ \gamma_{\rho} \hat{p}_1 \gamma_{\rho} e.$$  

Fig. 1. Diagrams contributing at the Born level.
where $\hat{e}$ ($p_i$) is the polarization vector of the real photon and $H_p(q)$ is the current describing the conversion of a virtual photon with momentum $q$ to a hadronic state. We restrict ourselves to the kinematic conditions of large-angle scattering,

$$s = 2p_+ p_-, \quad \chi_+ = 2p_+ p_\perp, \quad p_\perp^2 = 0,$$

$$p_\perp^2 = m^2, \quad s - \chi_+ - \chi_- = q^2, \quad q^2 > 0,$$

$$s \sim q^2, \quad \chi_+ \sim \chi_- \gg m^2. \quad (3)$$

In the expressions below, we set $m = 0$ everywhere except the denominators of loop integrals.

The cross section can be expressed in terms of the modulus of a squared matrix element summed over spin states:

$$\sum_{\text{spin}} |M|^2 = (4\pi\alpha)^3 B_{pp_1}^B H_{pp_1}^B \left(\frac{q^2}{\omega}\right)^2,$$

$$B_{pp_1}^B = \frac{1}{4} \text{Tr} \rho \hat{O}_{p_1} \rho \hat{O}_{p_1}, \quad H_{pp_1}^B = \sum_{\text{spin}} H_p(q) H_p^B(q). \quad (4)$$

The differential cross section can be written as

$$d\sigma_{e^- e^+ \rightarrow \gamma X} = \frac{1}{8s} \sum_{\text{spin}} |M|^2 \frac{d^3 p_1}{2\omega(2\pi)} d\Gamma_f,$$

$$d\Gamma_f = (2\pi)^4 \delta^4 \left( p_+ - p_+ - p_1 - \sum_{f} q_i \right) \prod_{f} \frac{d^3 q_i}{2e_i(2\pi)^3}.$$

For the differential hard-photon cross section, we obtain

$$\omega_1 d\sigma_{e^- e^+ \rightarrow \gamma X} = \frac{2\alpha^3}{s \left(\frac{q^2}{\omega}\right)^2} H_{pp_1}^B B_{pp_1}^B, \quad (5)$$

where

$$B_{pp_1}^B = B_{p_1} \tilde{g}_{pp_1} \rho + B_{\rho_+ \rho_+ \rho_- \rho_-} + B_{-\rho_- \rho_- \rho_+ \rho_+} + B_{\rho_+ (\rho_+ \rho_+ \rho_- \rho_-)} + B_{\rho_+(\rho_+ \rho_+ \rho_- \rho_-)} \rho \rho_1,$$

$$B_{\rho_+ \rho_+ \rho_- \rho_-} = p_+ \rho_+ \rho_- \rho_+ + p_{\rho_+ \rho_+ \rho_- \rho_-} \rho^- \rho^-.$$

The quantities with the “tilde” are defined as

$$\bar{g}_{\rho_1} = g_{\rho_1} - \frac{1}{q^2}\bar{p}_{\rho_1} \bar{q}_{\rho_1}, \quad \bar{p}_{\rho_1} = p_{\rho_1} - \frac{q^2}{q^2}, \quad (6)$$

In the Born approximation (see Fig. 1), we have

$$B^B_{\rho_+} = \frac{1}{\chi_+ \chi_-} (2\beta_1^2 + \chi_+^2 + \chi_-^2),$$

$$B^B_{\rho_+} = 4\beta_1^2, \quad B^B = 0. \quad (8)$$

For $q^2 = 0$, we reproduce the Dirac cross section of $e^- e^+ \rightarrow \gamma X$:

$$\frac{d\Gamma}{d\Omega_1} = \frac{2\alpha^2 \chi_+^2 + \chi_-^2}{s \chi_+ \chi_-}. \quad (9)$$

Below, we concentrate on the calculation of the one-loop radiative correction to the Dirac tensor.

We show that in considering the corrections, only a half of the full set of Feynman diagrams for process (2) can be used. We set

$$O_\rho = O^- \rho + O^+ \rho,$$

separating the contribution of emission from the electron leg $O^- \rho$ and the positron one $O^+ \rho$ (see Fig. 1 for the Born case and Fig. 2 for the one-loop corrections).

It can be shown that using the cyclic property of the trace and the mirror property

$$\text{Tr} \hat{a}_1 \hat{a}_2 \ldots \hat{a}_n = \text{Tr} \hat{a}_n \hat{a}_1 \ldots \hat{a}_2,$$

the total contribution to the Dirac leptonic tensor can be written as

$$\text{Tr} \hat{\rho} \alpha_p \alpha_p \omega \hat{O}_\rho \omega \hat{O}_\rho + \text{Tr} \hat{\rho} \alpha_p \omega \hat{O}_\rho \omega \hat{O}_\rho \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p,$$

$$= (1 + \Delta_{pp_1})(1 + \hat{\rho}) \text{Tr} \hat{\rho} \alpha_p \omega \hat{O}_\rho \omega \hat{O}_\rho \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p \hat{\alpha}_p. \quad (10)$$