Bianchi Type-I Cosmology in $f(R, T)$ Gravity

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Abstract—We investigate the exact solutions of a Bianchi type-I space–time in the context of $f(R, T)$ gravity [1], where $f(R, T)$ is an arbitrary function of the Ricci scalar $R$ and the trace of the energy–momentum tensor $T$. For this purpose, we find two exact solutions using the assumption of a constant deceleration parameter and the variation law of the Hubble parameter. The obtained solutions correspond to two different models of the Universe. The physical behavior of these models is also discussed.

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1. INTRODUCTION

The most popular issue in the modern-day cosmology is the current expansion of the Universe. It is now evident from observational and theoretical facts that our universe is in the phase of accelerated expansion [2–10]. The phenomenon of dark energy and dark matter is another topic of discussion [11–18]. It was Einstein who first proposed the concept of dark energy and introduced a small positive cosmological constant. But after some time, he referred to it as the biggest mistake in his life. However, it is now believed that the cosmological constant may be a suitable candidate for dark energy. Another proposal to justify the current expansion of the Universe comes from modified or alternative theories of gravity. The $f(T)$ theory of gravity is one such example that has been recently developed. This theory is a generalized version of teleparallel gravity in which the Weitzenböck connection is used instead of the Levi-Civita connection. The interesting feature of the theory is that it may explain the current acceleration without involving dark energy. A considerable amount of work has been done in this theory so far [19]. Another interesting modified theory is the $f(R)$ theory of gravity involving a general function of the Ricci scalar in the standard Einstein–Hilbert Lagrangian. Some review articles [20] can be helpful in understanding the theory.

Many authors have investigated $f(R)$ gravity in different contexts [21–34]. Spherically symmetric solutions are most commonly studied solutions due to their closeness to Nature. Vacuum and perfect fluid solutions of a spherically symmetric spacetime in the metric version of this theory were explored in [35]. They used the assumption of a constant scalar curvature and found that the solutions corresponded to the already existing solutions in general relativity (GR). Noether symmetries have been used in [36] to study spherically symmetric solutions in $f(R)$ gravity. Similarly, many interesting results have been found using spherical symmetry in $f(R)$ gravity [37]. Cylindrically symmetric vacuum and nonvacuum solutions have also been explored in this theory [38]. Plane symmetric solutions were found in [39]. The same authors [40] discussed the solutions of Bianchi type-I and V cosmologies for vacuum and nonvacuum cases. Conserved quantities in $f(R)$ gravity via the Noether symmetry approach were recently calculated in [41].

In a recent paper [1], a new generalized theory known as $f(R, T)$ gravity was proposed. In this theory, gravitational Lagrangian involves an arbitrary function of the scalar curvature $R$ and the trace of the energy–momentum tensor $T$. The laws of thermodynamics in this theory were studied in [42]. The same authors [43] investigated holographic and agegraphic $f(R, T)$ models. In [44], $f(R, T)$ gravity was reconstructed by taking

$$f(R, T) = f_1(R) + f_2(T),$$

and it was proved that $f(R, T)$ gravity allows transition from matter-dominated phase to an acceleration phase. Thus, it is hoped that $f(R, T)$ gravity may explain the recent phase of cosmic acceleration of our Universe. This theory can be used to explore many issues and may provide some satisfactory results.

The isotropic models are considered to be most suitable to study the large-scale structure of the Universe. However, it is believed that the early Universe may not have been exactly uniform. This prediction motivates us to describe the early stages of the Universe with the models having an anisotropic background. Thus, the existence of anisotropy in early phases of the Universe is an interesting phenomenon to investigate. A Bianchi type-I cosmological model, being a generalization of the flat Friedmann–Robertson–Walker (FRW) model, is one of the simplest models of the anisotropic Universe. Therefore, it seems interesting to explore Bianchi-type models in

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the context of $f(R, T)$ gravity. Exact solutions of the $f(R, T)$ field equations for a locally rotationally symmetric Bianchi type-I space–time were investigated in [45]. Solutions of a Bianchi type-III spacetime were explored in [46] using the law of variation of Hubble’s parameter. Bianchi type-III dark energy model in the presence of a perfect fluid source has been reported [47]. Bianchi type-V cosmology in this theory was studied in [48] by involving the cosmological constant [47]. Solutions of the Bianchi type-V bulk viscous string cosmological model, were given in [49].

In this paper, we focus on investigating the exact solutions of a Bianchi type-I spacetime in the framework of $f(R, T)$ gravity. The plan of the paper is as follows. In Sec. 2, we give some basics of $f(R, T)$ gravity. Section 3 provides the exact solutions for a Bianchi type-I spacetime. Concluding remarks are given in the last section.

2. SOME BASICS OF $f(R, T)$ GRAVITY

The $f(R, T)$ theory of gravity is a generalization or modification of GR. The action for this theory is given [1]

$$S = \int -g \left( \frac{1}{16\pi G} f(R, T) + L_m \right) d^4x,$$  

(1)

where $f(R, T)$ is an arbitrary function of the Ricci scalar $R$ and the trace $T$ of the energy–momentum tensor $T_{\mu\nu}$, and $L_m$ is the usual matter Lagrangian. It is worth mentioning that if we replace $f(R, T)$ with $f(R)$, we obtain the action for $f(R)$ gravity, and the replacement of $f(R, T)$ with $R$ leads to the GR action. The energy–momentum tensor $T_{\mu\nu}$ is defined as [50]

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(-g L_m)}{\delta g^{\mu\nu}}.$$  

(2)

We assume that the dependence of the matter Lagrangian is merely on the metric tensor $g_{\mu\nu}$ rather than on its derivatives. In this case, we obtain

$$T_{\mu\nu} = L_m g_{\mu\nu} - 2 \frac{\delta L_m}{\delta g^{\mu\nu}}.$$  

(3)

The $f(R, T)$ gravity field equations are obtained by varying the action $S$ in Eq. (1) with respect to the metric tensor $g_{\mu\nu}$:

$$f_R(R, T) R_{\mu\nu} - \frac{1}{2} f(R, T) g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu}) \Theta = 0,$$  

(4)

where $\nabla_\mu$ denotes the covariant derivative and

$$\Box \equiv \nabla^\mu \nabla_\mu,$$  

$$f_R(R, T) = \frac{\partial f_R(R, T)}{\partial R},$$  

$$f_R(R, T) = \frac{\partial f_R(R, T)}{\partial T},$$  

$$\Theta_{\mu\nu} = g^{\alpha\beta} \delta T_{\alpha\beta}.$$  

Contraction of (4) yields

$$f_R(R, T) R + 3 \Box f_R(R, T) - 2 f(R, T) = \kappa T - f_T(R, T)(T + \Theta),$$  

(5)

where

$$\Theta = \Theta_{\mu}^\mu.$$  

This is an important equation because it provides a relation between the Ricci scalar $R$ and the trace $T$ of the energy–momentum tensor. Using the matter Lagrangian $L_m$, the standard matter energy–momentum tensor is derived as

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu},$$  

(6)

where

$$u_\mu = \sqrt{-g} (1, 0, 0, 0)$$  

is the four-velocity in comoving coordinates and $\rho$ and $p$ respectively denote the energy density and pressure of the fluid. Perfect-fluid problems involving energy density and pressure are not easy tasks. Moreover, there does not exist any unique definition for the matter Lagrangian. We can assume the matter Lagrangian $L_m = -p$, which gives

$$\Theta_{\mu\nu} = -pg_{\mu\nu} - 2 T_{\mu\nu},$$  

(7)

and consequently field equations (4) take the form

$$f_R(R, T) R_{\mu\nu} - \frac{1}{2} f(R, T) g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu}) \Theta = 0,$$  

(8)

We note that these field equations depend on the physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R, T)$ gravity are possible. However, three classes of these models were given in [1]:

$$f(R, T) = \begin{cases} R + 2 f(T), \
 f_1(R) + f_2(R), \
 f_3(R). \end{cases}$$  

In this paper, we focus on the first class, i.e.,

$$f(R, T) = R + 2 f(T).$$