Radiation of Gas Layer over Hot Surface

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Abstract—A method is presented for evaluation the radiation flux produced by a gas layer near a heated surface, where the gas temperature depends on a distance from the surface. This method refers to small temperature gradients and operates with an effective radiation temperature for each frequency, as well as with the width of the gas absorption band. These parameters are determined by the absorption spectrum of atoms or gas molecules, and also by the shape of the spectral line for the radiative transition between certain states of atomic particles of a gas. The possibilities of this method are demonstrated by examples of emission of photons from the solar photosphere, as well as emission of CO₂ molecules in the atmospheres of the Earth and Venus.

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1. INTRODUCTION

The radiative action of a gas near a hard hot surface is manifested in various physical situations, contributing to the energy balance of the surface. We give examples of this kind. Consider a spacecraft or a large meteorite entering the dense layers of the atmosphere. Then the object is heated under the action of frictional forces resulted from interaction with air streams which flow around its surface. In this case, the energy balance of the surface of the apparatus is determined by its heating under the action of air streams passing around it and by radiation of the surface. A high temperature of the surface leads to its partial evaporation, and the evaporated vapor, like heated air near the surface, absorbs radiation from the surface in a certain spectrum range and partially returns it to the surface. Thus, the evaporated material near the surface, like heated air, affects the energy balance of the moving object.

A classic example of this kind refers to the greenhouse effect, the nature of which was explained by Fourier in the early 19th century [1, 2] and which is reduced to a partial screening of the surface radiation of a gas located above it. In this case, gas molecules partially absorb the radiation of the surface and return it back. The examples considered below for emission of a gas above the surface refer to the greenhouse effect of planets, namely, Venus and the Earth, due to the emission of molecules of carbon dioxide. The absorption spectrum of carbon dioxide molecules is convenient for demonstration of the greenhouse effect, since, on the one hand, it has a regular structure, which simplifies and makes visual operations with it, and, on the other hand, it is quite complex, since it includes a large number of rotational transitions and several vibrational transitions. These examples demonstrate a variety of physical situations and convince us of the inadvisability of using universal programs in the analysis of real situations of this type.

2. RADIATION OF PLANE LAYER OF EQUILIBRIUM GAS

Our goal is to determine the radiation flux above a hard surface with participation of the surface and the gas above it, both a partial radiation flux at a given frequency, and the flux integrated over frequencies in the absorption spectrum range of gas molecules. A characteristic of interaction between radiation and gas is the absorption coefficient of the gas $k_\omega$ at a given frequency $\omega$, so that the variation in the energy flux $I_\omega$ for radiation at a given frequency during propagation through a gas in a direction $z$ is determined by the Beer—Lambert law [3, 4]

$$\frac{dI_\omega}{dz} = -k_\omega I_\omega$$  (2.1)

From this it follows that the probability of survival for a photon $P_\omega$ of a given frequency $\omega$, if it passes a certain distance $L$ from a point of the surface, is

$$P_\omega = \exp\left(-\int k_\omega dl\right),$$  (2.2)

where $dl$ is the element of the photon path from the surface. Below we consider the widespread case, where the absorption coefficient of a gas $k_\omega$ depends only on a distance from the surface.
$$P_\omega = \exp \left( -\frac{L}{\cos \theta} k_\omega dz \right) = \exp \left( -\frac{u_\omega}{\cos \theta} \right), \quad (2.3)$$

where the $z$ axis is perpendicular to the surface, $L$ is the total thickness of the layer, $\theta$ is the angle between the trajectory of the photon and the perpendicular to the surface, and $u_\omega$ is the optical thickness of the layer,

$$u_\omega = \int_0^L k_\omega dz. \quad (2.4)$$

Namely the optical thickness of the gas layer is the parameter characterizing the process under consideration.

Next, we determine the radiation flux $j_\omega$ at a given frequency from the surface of a plane layer, which is the kernel of the Biberman–Holdstein equation [5, 6] for radiation transfer in a medium and is given by

$$j_\omega = \int a_\omega N_\omega (r) dr \frac{1}{\tau}, \quad (2.5)$$

Here the point of the radiation exit is the origin, $a_\omega$ is the distribution function of emitted photons over frequencies, that is normalized according to the relation

$$\int a_\omega d\omega = 1,$$

the number of emission events per unit time is $N_\omega (r)/\tau$, so that $N_\omega (r)$ is the number of atomic particles per unit volume in the upper state of the radiative transition, $\tau$ is the lifetime of the upper transition state with respect to radiation, $4\pi r^2$ is the surface area which is crossed by photons at isotropic radiation, and the last factor, as before, is the probability of photon survival in the course of its movement to the layer surface.

In the case where the gas temperature $T$ is constant over a gas layer of a thickness $L$, the flux of photons leaving its surface is (for example, [7, 8])

$$j_\omega = i_\omega [1 - 2E_\omega (u_\omega)], \quad E_\omega (x) = \int_1^\infty e^{-t} \frac{dt}{t}, \quad (2.6)$$

where $i_\omega$ is the flux of photons from the surface of a blackbody, that is equal to

$$i_\omega = \frac{a_\omega N_\omega}{4k_\omega \tau} = \frac{\omega^2}{4\pi^2 c^2 \left[ \exp (\hbar \omega / T) - 1 \right]} \quad (2.7)$$

Below instead of the photon flux $j_\omega$ we will deal with the energy flux transported by photons $j_\omega = \hbar \omega i_\omega$. This energy flux for a blackbody $I_\omega = \hbar \omega i_\omega$ is (for example, [9, 10])

$$I_\omega = \hbar \omega i_\omega = \frac{\hbar \omega^3}{4\pi^2 c^2 \left[ \exp (\hbar \omega / T) - 1 \right]}. \quad (2.8)$$

Note that using the thermodynamically equilibrium value for the density of excited atomic particles $N_\omega (r)$ in deriving formula (2.6), we assumed that the radiative processes do not affect the distribution over excited states. This holds true if the following criterion is fulfilled

$$N_\omega \kappa_{\text{rel}} \gg \frac{1}{\tau}, \quad (2.9)$$

where $N_\omega$ is the number density of atoms or gas molecules, $\kappa_{\text{rel}}$ is the rate constant of relaxation, i.e. of the collision transition from the upper state of the radiative transition to the lower one. In particular, for the radiative transition considered below from the lower excited vibrational state of the carbon dioxide molecule $01^0$ to the ground state with a wavelength of about 15 μm, the relaxation rate constant of the vibrationally excited state at room temperature in air is [11] $\kappa_{\text{rel}} = 5 \times 10^{-16} \text{ cm}^3/\text{s}$, and requires for the number density of air molecules $N_\omega \gg 3 \times 10^{13} \text{ cm}^{-3}$. This is valid, in any case, for the troposphere and requires for the air pressure $p \gg 0.1 \text{ Torr}$. If the criterion (2.9) is fulfilled, the formation of an excited atomic states in a gas upon absorption of a photon is “forgotten” as a result of collisional processes, which also create excitations in the gas and thereby determine the emission of radiation from the gas.

### 3. RADIATION OF AN INHOMOGENEOUS FLAT GAS LAYER

Let us spread the results presented to a more general case, where a local thermodynamic equilibrium takes place in the gas, i.e., the state of a gas at a certain point can be characterized by the gaseous temperature at this point. Considering the radiation of a plane gas layer, we will assume that the temperature of the gas above the surface depends only on a distance from it. This assumption is more or less satisfied for extended objects, in particular, for the atmosphere of planets and stars, as well as for the gas flowing around the object moving in the atmosphere. Thus, we set the problem as follows. A gas is located above a flat surface, and the gas temperature varies with a distance $z$ from it. Introducing as a variable the optical thickness of the layer $u_\omega$ instead of a distance $z$ from the surface, we have for a photon flux of a given frequency crossing the layer boundary

$$j_\omega = \frac{\omega^2}{8\pi^2 c^2 \tau} \int \cos \theta d\omega \int_0^u \exp (\hbar \omega / T) F(u_\omega), \quad (3.1)$$

$$u_\omega (z) = \int_0^z k_\omega dz, \quad F(u_\omega) = \left( \exp \frac{\hbar \omega}{T(z)} - 1 \right)^{-1},$$

where $u_\omega$ is the total optical thickness of the layer at a given frequency. This formula follows from expressions (2.5) and (2.6) for equilibrium of radiation fluxes in the case of a constant temperature over the layer. Next, we will use this formula in the analysis of specific physical situations associated with emission of a gas layer.