\[ B_d(\bar{B}_d) \rightarrow \rho^\pm \pi^\mp, \rho^+ \rho^-, \pi^+ \pi^- : \text{Hunting for Alpha}^* \]

M. I. Vysotsky

Institute of Theoretical and Experimental Physics,
Boi’shaya Cheremushkinskaya ul. 25, Moscow, 117259 Russia

Received September 7, 2005

Abstract—We determine the domains of the values of unitarity-triangle angle \( \alpha \) allowed by charmless strangless \( B_d(\bar{B}_d) \) decays.

PACS numbers: 13.20.Jf, 13.30.Eg

DOI: 10.1134/S1063778806040119

1. INTRODUCTION

In paper [1], from the data on \( CP \) asymmetries in \( B_d(\bar{B}_d) \rightarrow \rho^\pm \pi^\mp, \rho^+ \rho^- \) and BABAR data on \( CP \) asymmetries in \( B_d(\bar{B}_d) \rightarrow \pi^+ \pi^- \) decays, we determine the value of angle \( \alpha \) of the unitarity triangle:

\[ \alpha = 96^\circ \pm 3^\circ, \]  

where only a tree quark decay amplitude of \( b \rightarrow u\bar{d}\bar{d} (b \rightarrow u\bar{d}d) \) was taken into account. The numerical values of angle \( \alpha \) obtained from the considered decays are consistent with each other and with the value which follows from the global CKM fit. This observation testifies to the validity of the proposed approach.

As the next step in the present paper, we will study what changes in the values of \( \alpha \) are induced by QCD penguins. Our aim is twofold. First, in this way, we will get an estimate of the theoretical uncertainty of the value of \( \alpha \) determined in [1]. Second, we will get the formulas for \( CP \)-violating parameters describing these decays which vanish when penguins are neglected (\( C_{\rho\pi}, A_{C\rho\pi}, C_{\rho\rho}, C_{\pi\pi} \)).

The angle shifts \( \Delta \alpha \) that we are interested in were estimated in [2]; however, in that paper, FSI phases were neglected; that is why \( C_{\rho\pi} = A_{C\rho\pi} = C_{\rho\rho} = C_{\pi\pi} = 0 \) follows from [2] (the nonzero penguin amplitudes are a necessary but not a sufficient condition for \( C_{\rho\pi}, \ldots \) to be nonzero). We will take these phases into account. The asymmetries depend on the differences of FSI phases in the processes described by the tree and penguin diagrams. One source of these differences is an imaginary part of the quark penguin diagram, the so-called BSS mechanism of the strong phase generation [3] (see also [4]). The phase of the penguin diagram depends on the gluon \( q^2 \) which is transferred to a \( u\bar{u} \) pair, each quark of which goes to different \( \pi^\pm \) or \( \rho^\pm \) mesons. In this way, the value of \( q^2 \) depends on the light-meson wave functions, and we can estimate it only roughly. Another source of FSI phases is hadron rescattering, and even less is known about the values of the phase shifts between the penguin and tree diagrams generated in this way. In view of this, we will determine FSI phases from the experimental data on \( CP \)-violating asymmetries and investigate to what values of \( \alpha \) it will lead.

In the Appendix, we present the weak-interaction Hamiltonian which is responsible for \( b \rightarrow u\bar{d}d \) transition and calculate the necessary matrix elements. Using these formulas in Sections 2, 3, and 4, we study \( B \rightarrow \rho\pi, \rho\rho, \) and \( \pi\pi \) decays, correspondingly, and extract the values of angle \( \alpha \) from the experimental data on \( CP \) asymmetries in these decays. We conclude in Section 5 with the averaged value of \( \alpha \) and a general discussion.

2. \( \alpha \) FROM \( \bar{B}_d(B_d) \rightarrow \rho^\mp \pi^\pm \)

The time dependence of the decay probabilities is given by [5]

\[ \frac{dN(B_d(\bar{B}_d) \rightarrow \rho^\pm \pi^\mp)}{dt} = \left( 1 \pm A_{C\rho\pi}^\pi \right) e^{-t/\tau} \left[ 1 - q(C_{\rho\pi} \pm \Delta C_{\rho\pi}) \times \cos(\Delta mt) + q(S_{\rho\pi} \pm \Delta S_{\rho\pi}) \sin(\Delta mt) \right], \]

where \( q = -1 \) describes the case where at \( t = 0 \) \( B_d \) was produced, while \( q = 1 \) corresponds to \( \bar{B}_d \) production at \( t = 0 \). In the case of \( \Upsilon(4S) \rightarrow B_d\bar{B}_d \) decay, the flavor of the beauty meson which will decay to \( \rho\pi \) is tagged by the charge of a lepton in the other beauty meson semileptonic decay. A partner decay starts the clock as well. \( \tau \) is \( B_d(\bar{B}_d) \) lifetime, while \( \Delta m \) is the
difference in masses of \((B_d, \bar{B}_d)\) system eigenstates (it equals the frequency of \(B_d - \bar{B}_d\) oscillations).

From Eqs. (A.15) and (A.17) in the Appendix, we obtain

\[
M^+ = AV_{ud}V_{ud}^*[1 - 0.07 e^{i(\delta - \alpha)}] = AV_{ud}V_{ud}^*[1 - 0.07 \sin \delta + 0.07 i \cos \delta],
\]

\[
M^- = BV_{ud}V_{ud}^*,
\]

\[
\lambda = \frac{q M^+}{p M^-} = e^{2i\alpha} \left[1 - 0.07 \sin \delta + 0.07 i \cos \delta\right],
\]

where parameters \(q\) and \(p\) enter the expressions for \((B_d, \bar{B}_d)\) eigenstates and we have substituted \(\alpha = \pi/2\) in the (small) second term in square brackets in Eq. (A.15). Analogously, we get

\[
M^+ = AV_{ud}V_{ud}^*[1 + 0.07 e^{i(\alpha + \delta)}] = AV_{ud}V_{ud}^*[1 + 0.07 \sin \delta - 0.07 i \cos \delta],
\]

\[
\lambda = \frac{q M^+}{p M^-} = e^{2i\alpha} B \left[1 + 0.07 \sin \delta - 0.07 i \cos \delta\right]^{-1}.
\]

From the expressions for the quantities \(C_{\rho\tau}\) and \(\Delta C_{\rho\tau}\) [5],

\[
C_{\rho\tau} \pm \Delta C_{\rho\tau} = \frac{1 - \lambda^\pm|2^2}{1 + \lambda^\pm|2^2},
\]

we obtain

\[
\Delta C_{\rho\tau} = \frac{a^2 - b^2}{a^2 + b^2}, \quad \frac{a^2}{b^2} = \frac{1 + \Delta C_{\rho\tau}}{1 - \Delta C_{\rho\tau}},
\]

\[
C_{\rho\tau} = 0.28 \sin \delta \frac{(a/b)^2}{(1 + a^2/b^2)^2},
\]

where \(a \equiv |A|,\) \(b \equiv |B|)\).

The Belle- and BABAR-averaged result for \(\Delta C_{\rho\tau}\) is [6]

\[
\Delta C_{\rho\tau} = 0.22 \pm 0.10,
\]

which leads to

\[
\left(\frac{a}{b}\right)^2 = 1.56 \pm 0.33.
\]

From the averaged experimental result [6]

\[
C_{\rho\tau} = 0.31 \pm 0.10
\]

and Eq. (10), we get

\[
\sin \delta = 4.6 \pm 1.5.
\]

We see that poor accuracy in the measurement of \(C_{\rho\tau}\) does not allow one to get any definite information on the value of phase \(\delta\).

The next observable that we wish to discuss is \(CP\) asymmetry \(A_{\rho\tau}^{CP}\):

\[
A_{\rho\tau}^{CP} = \frac{|M^+|^2 - |M^-|^2 + |M^+|2 - |M^-|^2}{|M^+|^2 + |M^-|^2 + |M^+|^2 + |M^-|^2} = 0.14 \sin \delta \frac{(a/b)^2}{1 + (a/b)^2},
\]

which should be compared with the experimental result [6]

\[
A_{\rho\tau}^{CP} = -0.102 \pm 0.045.
\]

From (15) and (16), we get

\[
\sin \delta = -1.2 \pm 0.5,
\]

and it differs from that given in Eq. (14) by 3.5 standard deviations. This is the largest discrepancy that we encounter in this paper. Averaging these two numbers, we obtain

\[
\sin \delta = -0.62 \pm 0.47.
\]

Finally, we come to the discussion of the observables which are sensitive to the angle \(\alpha\):

\[
S_{\rho\tau} \pm \Delta S_{\rho\tau} = \frac{2 \lambda\sin(2\alpha) \cos \tilde{\delta}}{1 + \lambda^\pm|2^2},
\]

\[
S_{\rho\tau} = \frac{2a/b}{1 + a^2/b^2} \left[\sin(2\alpha) \cos \tilde{\delta} - 0.07 \cos \delta \cos \tilde{\delta} - 0.07 \cos \delta \sin \tilde{\delta} \sin(2\alpha) + 0.07 \cos \delta \sin \tilde{\delta} \sin(2\alpha) \right],
\]

\[
\Delta S_{\rho\tau} = \frac{2a/b}{1 + a^2/b^2} \left[-\cos(2\alpha) \sin \tilde{\delta} + 0.07 \cos \delta \sin \tilde{\delta} \sin(2\alpha) \right],
\]

where the definition of the phase \(\tilde{\delta}\) is \(A/B \equiv (a/b)\sin \tilde{\delta}\) and in the small terms proportional to 0.07 in the expression for \(S_{\rho\tau}\) we have substituted \(\cos(2\alpha) = -1\).

Let us start the analysis of the experimental data from \(\Delta S_{\rho\tau}\). According to [6]

\[
\Delta S_{\rho\tau} = 0.09 \pm 0.13,
\]

which is much less than one. According to Eq. (12), the factor which multiplies the expression in square brackets in Eq. (20) [and in Eq. (21)] is very close to one; that is why it is the expression in square brackets which should be much less than one. The second and