General Covariance Violation and the Gravitational Dark Matter:
Scalar Graviton*

Yu. F. Pirogov

Institute for High Energy Physics, Protvino, Moscow oblast, 142284 Russia

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Abstract—The violation of the general covariance is proposed as a resource of the gravitational dark matter. The minimal violation of the covariance to the unimodular one is associated with the massive scalar graviton as the simplest representative of such matter. The Lagrangian formalism for a continuous medium, a perfect fluid in particular, in the scalar graviton environment is developed. The implications for cosmology are briefly indicated.

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1. INTRODUCTION

General Relativity (GR) is the viable theory of gravity, perfectly consistent with observations. Nevertheless, it encounters a number of conceptual problems, among which one can mention that of the dark matter (DM). To solve the latter problem, one usually adjusts the conventional or new matter particles, remaining still in the realm of GR (for a recent review, see, e.g., [1]). The ultimate goal of the DM being in essence to participate only in gravitational interactions, one can try to attribute to this purpose the gravity itself, thus going beyond GR.

Namely, GR is the field theory of the massless tensor graviton as a part of the metric. However, the metric also contains extra degrees of freedom, which could be associated with the scalar, vector, and tensor gravitons. Nevertheless, all the extra degrees of freedom lack the explicit physical manifestations. This is due to the fact that GR incorporates as the basic symmetry the general covariance (GC). The latter serves as the gauge symmetry to remove from the observables the degrees of freedom in excess of the massless tensor graviton. So, to associate the extra particles contained in the metric with (a part of) the DM, the violation of the GC is obligatory.

Starting from GR, an arbitrary variety of GC violations is admissible. Contrary to this, a hierarchy of GC violations is expected in the framework of the affine Goldstone approach to gravity [2]. The latter approach, in contrast to GR, is based on two symmetries: the global affine symmetry (AS) plus the GC. As a result, there are two conceivable types of GC violations: those with and without explicit AS violation. The first type of violation is to be strongly suppressed. But the second one is a priori arbitrary. Under reasonable assumptions, the GC violation can be parametrized in the affine Goldstone approach through the background metric.1)

Marginally, the latter description can depend only on the determinant of the background metric. In this case, there still resides the unimodular covariance (UC). In comparison with the GC, the UC lacks only the local scale transformations. Being next-order to the GC, the UC suffices to incorporate both the massive scalar graviton and the massless tensor one, but nothing redundant.2) The violation of the GC to the residual UC is considered in the present paper to be the raison d’etre for the existence of the scalar graviton as simplest representative of the gravitational DM. More elaborate dependences on the background metric, with the residual covariance group being even more restricted, could correspond to the rest of the metric degrees of freedom.

In the present paper, we confine ourselves to the scalar graviton, vector and tensor ones being postponed to the future. In Section 2, the GC violation, in general, and, specifically, the scalar graviton are considered. The Lagrangian description of the continuous medium, the perfect fluid in particular, in the scalar graviton environment is developed in Section 3, with some remarks in Conclusions.

1) The basic ingredient of the approach is the spontaneous appearence of the metric in the affinely connected world. For a short exposition, see [3]. For a related development, see [4].

2) In fact, it is the UC, not the GC, which ensures the masslessness of the tensor graviton [5]. In other contexts, the UC is studied in [6–9].
2. GRAVITY

Gravitational DM

Let $x^\mu$, $\mu = 0, \ldots, 3$, be an arbitrary observer’s coordinates. Let us consider the classical theory of the metric $g_{\mu\nu}$ and the generic matter field $\phi$ with the action

$$I = \int \left( L_g(g_{\mu\nu}) + \Delta L_g(g_{\mu\nu}, \xi_A) \right) \sqrt{-g} \, d^4x.$$  

Here $L_g$ and $\Delta L_g$ are, respectively, the generally covariant and GC-violating contributions of the gravity, $L_m$ being the matter Lagrangian. In general, the latter violates the GC too. All the Lagrangians above are assumed to be scalars. The background fields $\xi_A = \xi_A(x^\mu)$, $A = 1, \ldots$, parametrize the GC violation and ultimately determine the group of the residual covariance (if any). These parameter fields are the generalization of the constant parameters and are to be found through observations.\(^3\) The spacetime dependence of $\xi_A$ tacitly implies the existence of distinguished coordinates relative to some physical background. This means that the metric Universe, contrary to what is assumed in GR, is not a self-contained system and cannot exclusively be described in internal dynamical terms.

It is well known that the generally covariant Lagrangian $L_g$ corresponds to the massless tensor graviton (see, e.g., [5]). The reason is that the GC removes in the equations of motion all the degrees of freedom contained in the metric but those with the helicities $\lambda = \pm 2$. To make the rest of the metric degrees of freedom observable, the addition of the GC-violating $\Delta L_g$ is obligatory.

Varying the action (1) with respect to $g^{\mu\nu}$, one arrives at the gravity equation of motion:

$$G_{\mu\nu} + \Delta G_{\mu\nu} = M_P^{-2} T_{\mu\nu}. \tag{2}$$

Here, $G_{\mu\nu}$ is the gravity tensor defined as follows:

$$G_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta L_g}{\delta g^{\mu\nu}}. \tag{3}$$

with $L_g = \sqrt{-g} L_{g}$ being the gravity Lagrangian density (and similarly for $\Delta G_{\mu\nu}$). In the above, one conventionally sets $M_P^{-2} = 8\pi G_N$, with $M_P$ being the Planck mass and $G_N$ being Newton’s constant. $T_{\mu\nu}$ is the conventional energy-momentum tensor of the matter defined through $T_m = \sqrt{-g} L_m$ similar to the right-hand side of Eq. (3).

Owing to the GC of $L_g$, one arrives at the identity

$$\nabla_\nu G^{\mu\nu} = 0, \tag{4}$$

with $\nabla_\nu$ being the covariant derivative defined by the metric $g_{\mu\nu}$. Thus, with account of Eq. (2), the modified conservation condition

$$\nabla_\nu (T^{\mu\nu}_m - M_P^2 \Delta G^{\mu\nu}) = 0 \tag{5}$$

is to be fulfilled. Clearly, the energy–momentum of ordinary matter alone ceases to conserve. To restore the conventional interpretation of conserved matter, one should consider the extra gravity degrees of freedom as (a part of) the DM with the energy–momentum tensor

$$\Delta T^{\mu\nu}_m = -M_P^2 \Delta G^{\mu\nu}. \tag{6}$$

The respective particles have no specific quantum numbers and possess only gravitational interactions, so that such an association is quite natural.

In other terms, Eq. (5) can be written as

$$\nabla_\nu T^{\mu\nu}_m = Q^\mu, \tag{7}$$

with the vector

$$Q^\mu = M_P^2 \nabla_\nu \Delta G^{\mu\nu}, \tag{8}$$

representing the extra gravity force acting on ordinary matter. This force reflects the nonfulfillment of the equivalence principle due to the GC violation. In the case when $L_m$ does not depend on $\xi_A$ and thus is generally covariant, one gets with account of the matter equation of motion

$$\nabla_\nu T^{\mu\nu}_m = Q^\mu = 0. \tag{9}$$

Scalar Graviton

In what follows, we restrict ourselves to the minimal violation of the GC with the scalar graviton alone as the simplest representative of the gravitational DM. Conventionally, let us take as $L_g$ the modified Einstein–Hilbert Lagrangian:

$$L_g = -M_P^2 \left( \frac{1}{2} R(g_{\mu\nu}) - \Lambda \right), \tag{10}$$

with $R$ being the Ricci scalar and $\Lambda$ being the cosmological constant (if any). Equation (10) describes the massless tensor graviton.

To incorporate additionally only the scalar graviton, the minimal GC violation with the residual UC is sufficient. To this end, we use as the respective field variable the relative metric scale $e^\chi = \sqrt{-g}/\sqrt{\bar{g}}$ or, otherwise,

$$\chi = \frac{1}{2} \ln \frac{g}{\bar{g}}. \tag{11}$$

Here, one sets $g = \det g_{\mu\nu}$ and $\bar{g} = \det \bar{g}_{\mu\nu}$, with $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}(\xi_A)$ being the dynamical and background