About Radiative Kaon Decay $K^+ \to \pi^+\pi^0\gamma^*$

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Abstract—With usage of the Low theorem, the general expression for the amplitude of radiative kaon decay $K^+ \to \pi^+\pi^0\gamma$ is determined. The possible reason of suppression for the branching ratio of kaon decay $K^+ \to \pi^+\pi^0$ is considered.

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At first, let us consider nonleptonic $K^+$ decay $K^+ \to \pi^+\pi^0$. Suppose that this decay is suppressed by the $\Delta I = 1/2$ rule. It is well known that the $K^+$ is pseudoscalar $0^-$ state. Therefore, two final pions should be in a state with zero angular momentum. This means that the full wave function of a system of two pions should be symmetrical. Consequently, two pions can be in a state with an isospin $I = 0$ or $I = 2$. In decay $K^+ \to \pi^+\pi^0$, the final state should have $I_3 = 1$. Then its isospin equals 2 with $I_3 = 1$. Thus, $\Delta I = 3/2$ or $\Delta I = 5/2$.

The matrix element for the process, shown by the diagram of Fig. 1,

$$K^+(p) \to \pi^0(q_1) + \pi^+(q_2),$$

where $p, q_1,$ and $q_2$ denote the four-momenta of the initial $K^+$ meson and the outgoing $\pi^0$ and $\pi^+$ mesons, respectively, is given by

$$T(K^+ \to \pi^0\pi^+) = \frac{G_F}{\sqrt{2}} \sin \theta_V \cos \theta_A \frac{1}{\sqrt{2}} (p + q_1)_\mu \alpha f_\pi g_\pi^\mu.$$  

Here, $G_F$ is the Fermi constant; $f_\pi$ is a charged-pion decay constant, $f_\pi = 0.131$ GeV [1]; $\theta_A$ and $\theta_V$ are the Cabibbo angles; $\alpha$ is a dimensionless hadronization factor introduced by us. The introduction of this factor is explained by the fact that not each quark–antiquark pair can form a meson, but only a pair with zero total spin and isospin $I = 1$. As a result, the amplitude (2) should be multiplied by Clebsch–Gordan coefficients on a spin and isospin $\alpha = (1/\sqrt{2})(1/\sqrt{2}) = 1/2$. The branching ratio of the decay $K^+ \to \pi^0\pi^+$, associated with amplitude (2), fits experimental data (21.13 ± 0.14)% [1] and (21.18 ± 0.28)% [2] at $\alpha \approx 0.5$.

The initial and final states of radiative $K^+ \to \pi^+\pi^0\gamma$ decay are not $CP$ eigenstates. Therefore, in the limit of $CP$ conservation, the amplitude of this decay is contributed by an internal bremsstrahlung (IB) and by the direct photon emission of the electrical and magnetic type. At the same time, the measurement of the interference of IB with electric dipole transitions is important for differentiating between various models of the description of $K^+ \to \pi^+\pi^0\gamma$ decay. Therefore, in the present article, the main attention is given to the revelation of direct vector contributions to the amplitude of $K^+ \to \pi^+\pi^0\gamma$ decay. In our approach, we will follow [3–7]. It is known that IB contributions have a logarithmic singularity in $k$; that is, upon integrating, they behave as $k^{-1}$ for $k \to 0$. We assume that direct vector contributions (DVC) and direct axial contributions (DAC) have no singularity for $k \to 0$, independently of how $k \to 0$.

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Diagram giving the $K^+$ inner bremsstrahlung term of $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ decay.

Diagram giving the $\pi^+$ inner bremsstrahlung term of $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ decay.

The $S$-matrix element of kaon decay

$$K^+(p) \rightarrow \pi^0(q_1) + \pi^+(q_2) + e(k)$$

is

$$\langle \pi^0, \pi^+, e|S|K^+ \rangle = i e (2\pi)^4 \delta(4) (q_1 + q_2) + k - p \varepsilon^\mu M_\mu(k),$$

where $\varepsilon^\mu$ is the photon polarization and $e$ is the proton electric charge. Taking into consideration the aforementioned, the quantity $M_\mu(k)$ can be written as

$$M_\mu(k) = M^{IB}_\mu(k) + M^{DVC}_\mu(k) + M^{DAC}_\mu(k).$$

Here, $M^{IB}_\mu(k)$ is the sum of terms corresponding to the photon emission from external charged lines (see Figs. 2, 3); $M^{DVC}_\mu(k)$ and $M^{DAC}_\mu(k)$ are the respective sums of direct emission (see Fig. 4). $M^{IB}_\mu(k)$ has contributions of the order of $k^{-1}$, $k^0$, and $O(k)$. The contributions of direct emission are of the order of $k^0$, $k^n$, $n \geq 1$.

From the electric-current conservation, it follows that

$$k^\mu M_\mu(k) = 0.$$  

Let us consider now the amplitude $T(P^2, Q^2_2)$ [5] for nonradiative off-mass-shell $K^+$ decay

$$K^+(P) \rightarrow \pi^0(Q_1) + \pi^+(Q_2).$$

This amplitude conserves momentum and energy but not mass. On the mass shell, the amplitude is

$$T(m_{K^+}^2, m_{\pi^0}^2, m_{\pi^+}^2).$$

For the nonradiative part of the process in Fig. 2, the amplitude is $T((p - k)^2, m_{\pi^0}^2, m_{\pi^+}^2)$. The amplitude corresponding to the process shown in Fig. 3 is $T(m_{K^+}^2, m_{\pi^0}^2, (q_2 + k)^2)$.

Accounting for these amplitudes, the amplitude $M^{IB}_\mu(k)$ may be written as

$$M^{IB}_\mu(k) = \left(\frac{2q_2 + k}{q_2 + k} - \frac{p_\mu}{p \cdot k}\right) \times T(m_{K^+}^2, m_{\pi^0}^2, m_{\pi^+}^2, (q_2 + k)^2)$$

$$+ T((p - k)^2, m_{\pi^0}^2, m_{\pi^+}^2, (2p - k)_\mu).$$

Expanding $M^{IB}_\mu(k)$ with respect to $k$ and using the conditions $k^2 = 0$ and $k \cdot e = 0$, we get

$$M^{IB}_\mu(k) = \frac{q_2 \cdot k}{q_2} - \frac{p_\mu}{p \cdot k} \times T(m_{K^+}^2, m_{\pi^0}^2, m_{\pi^+}^2)$$

$$+ 2q_2 \mu \frac{\partial T(m_{K^+}^2, m_{\pi^0}^2, m_{\pi^+}^2)}{\partial Q^2_2} \bigg|_{Q^2_2 = m_{\pi^+}^2}$$

$$+ 2p_\mu \frac{\partial T(P^2, m_{\pi^0}^2, m_{\pi^+}^2)}{\partial P^2} \bigg|_{P^2 = m_{K^+}^2} + O(k).$$

From Eqs. (5) and (6), we obtain

$$M^{DVC}_\mu(0) = M^{DAC}_\mu(0)$$

$$= -2q_2 \mu \frac{\partial T}{\partial Q^2_2} \bigg|_{Q^2_2 = m_{\pi^+}^2} - 2p_\mu \frac{\partial T}{\partial P^2} \bigg|_{P^2 = m_{K^+}^2}.$$