Quark Distributions in QCD Sum Rules: Unexpected Features and Paradoxes

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Abstract—Some very unusual features of the hadron structure functions, obtained in generalized QCD sum rules, like the surprisingly strong difference between longitudinally and transversally polarized \( \rho \)-meson structure functions and the strong suppression of the gluon sea in the longitudinally polarized \( \rho \) meson, are discussed.

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It is well known that QCD DGLAP equations predict the evolution of quark distributions with \( Q^2 \), but not the initial values at some relatively low \( Q^2 \) from which this evolution starts. To obtain these initial values (inputs), a form of quark (valence and sea) and gluon distributions with some free parameters is usually assumed. Then, by using DGLAP equations, quark and gluon distributions are calculated at all \( Q^2 \) and \( x \) and compared with the whole set of the experimental data, and in this way the initial parameters are fixed. Evidently, such an approach is not completely satisfactory from a theoretical point of view, because in this way one can lose some unexpected physical features of distributions, so it would be desirable to determine the initial distribution directly from QCD. It the present paper, we discuss how it is possible to find quark and gluon distributions in hadrons in a model-independent way in QCD, and what unexpected features of quark distributions then really appear.

The method to determine valence-quark distributions in QCD sum rules at intermediate \( x \) was suggested in [1] and generalized in [2, 3]. Owing to this generalization, it became possible to calculate valence-quark distributions in pions [2] and transversally and longitudinally polarized \( \rho \) mesons [3], and also significantly improve the result for valence-quark distributions in protons [4].

Let us at first briefly review the method. We start from consideration of the four-point correlator corresponding to the forward scattering of hadron and electromagnetic currents:

\[
\Pi = -i \int d^4xd^4yd^4ze^{ip_1x+iqy-ip_2z} \times \langle 0 | T \{ j^h(x), j^{\ast d}(y), j^{\ast d}(0), j^h(z) \} | 0 \rangle.
\]

Here, \( p_1 \) and \( p_2 \) are the initial and final momenta carried by hadronic current \( j^h \), \( q \) and \( q' = q + p_1 - p_2 \) are the initial and final momenta carried by virtual photons (Lorentz indices are omitted), and \( t = (p_1 - p_2)^2 = 0 \).

The idea of the approach (in the improved version) is to consider the imaginary part (in \( s \) channel) of this four-point correlator \( \Pi(p_1, p_2; q, q') \). It is supposed that virtualities of the photon \( q^2, q'^2 \) and hadron currents \( p_1^2, p_2^2 \) are large and negative:

\[
|q^2| = |q'^2| \gg |p_1^2|, |p_2^2| \gg R_c^{-2},
\]

where \( R_c \) is the confinement radius. It was shown in [1] that, in this case, the imaginary part in the \( s \) channel \( [s = (p_1 + q)^2] \) of \( \Pi(p_1, p_2; q_1, q') \) is dominated by a small distance contribution at not small \( x \), so operator product expansion (OPE) is applicable.

To find the structure function, one should compare the dispersion representation of the forward scattering amplitude in terms of physical states with those in OPE and use Borel transformation. Though we should consider the case of the forward scattering, i.e., \( p_1 = p_2 \), but, and it is a significant point of the method, we should keep \( p_1^2 \) not equal to \( p_2^2 \) in all intermediate stages and only in the final result take the forward-scattering limit. Only in such a way is it possible to effectively (exponentially) suppress all the terms in the sum rules, except for those which

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(ii) Power corrections—higher order terms of OPE. Among them, first of all, we should take into account the dimension-4 correction, proportional to the gluon condensate \( \langle 0 | \alpha_s^{n} G_{\mu\nu}^{a} G_{\alpha\beta}^{b} G_{\rho\sigma}^{c} | 0 \rangle \). The corresponding diagrams are shown in Fig. 2. But, surprisingly, it was found that these diagrams’ contribution to the sum rule is exactly canceled after Borel transformation on \( p_{\Gamma}^2 \) and \( p_{\Pi}^2 \). It should be noted that the diagrams in total may not cancel; cancellation takes place only for those parts of them which have a double-imaginary part (both on \( p_{\Gamma}^2 \) and \( p_{\Pi}^2 \), i.e., just those which contribute to the pion structure function). So we see that there is no gluon condensate contribution to the sum rule for the pion structure function. The fact of exact cancellation of contributions of ten diagrams in Fig. 2 is surprising, but of course one can treat this as just an accident. But just the same takes place for operators of the next dimension!

There are a large number of loop diagrams, corresponding to \( d = 6 \) corrections. First of all, there are diagrams which correspond to interaction only with the gluon vacuum field, i.e., only with external soft gluon lines (see Fig. 3). They are, of course, proportional to \( \langle g^3 f^{abc} G_{\mu\nu}^{a} G_{\alpha\beta}^{b} G_{\rho\sigma}^{c} \rangle \) and proportional to \( \langle 0 | D_{\mu} G_{\mu\nu}^{a} D_{\sigma} G_{\alpha\beta}^{b} | 0 \rangle \) and \( \langle 0 | G_{\mu\nu}^{a} G_{\rho\sigma}^{b} D_{\nu} D_{\mu} G_{\alpha\beta}^{c} | 0 \rangle \). Using the equation of motion, these vacuum averages can be expressed in terms of \( \langle 0 | g \bar{\psi} \psi | 0 \rangle \) and \( \langle 0 | g^3 c^{abc} G_{\mu\nu}^{a} G_{\rho\sigma}^{b} G_{\mu\nu}^{c} f^{abc} | 0 \rangle \).

As was shown in [2], if one takes the sum of all diagrams then the terms, proportional to the gluon condensate \( \langle 0 | g^3 c^{abc} G_{\mu\nu}^{a} G_{\rho\sigma}^{b} G_{\mu\nu}^{c} f^{abc} | 0 \rangle \) exactly cancel each other in just the same way as for the \( d = 4 \) operator we have discussed above. It should be noted that there are about a hundred different diagrams, so it is hard to believe that such a cancellation is again an accident.

So we can fix first the very unexpected feature of pion inner structure: the gluon vacuum contribution to the pion valence-quark distribution is exactly zero for dimensions 4 and 6. It is very likely that such a cancellation is a consequence of some new symmetry, though today we cannot understand it.

Of course, except for the diagrams in Figs. 3 and 4, there are other diagrams of dimension 6, directly proportional to the four-quark operators

\[ a^2 = \alpha_s(2\pi)^4 \langle \langle 0 | \bar{\psi} \psi | 0 \rangle \rangle^2. \]

We will not discuss them here; one can see papers [2, 3]. Finally, the quark distribution function has the following form:

\[ x u(x) = \frac{3}{2\pi} \frac{M^2}{f_\pi^2} 3^2 (1 - x) \]