Analysis of the Production of Higgs Boson Pairs at the One-Loop Level in the Minimal Supersymmetric Standard Model

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Received March 21, 2008

Abstract—Within the minimal supersymmetric standard model, the amplitudes and total cross sections for the processes \( e^+e^- \rightarrow hh, e^+e^- \rightarrow hH, e^+e^- \rightarrow HH, \) and \( e^+e^- \rightarrow AA \) are calculated in the first order of perturbation theory with allowance for a complete set of one-loop diagrams in the \( m_e \rightarrow 0 \) approximation. Analytic expressions are obtained for the quantities under consideration; numerical results are presented in a graphical form. It is shown that the cross section for the process \( e^+e^- \rightarrow hh \) is larger than those for the other processes (and is on the same order of magnitude as the cross section for the corresponding processes in the Standard Model). In the case of the collision energy equal to \( \sqrt{s} = 500 \) GeV, an integrated luminosity in the region \( \int L \geq 500 \text{ fb}^{-1} \), and a longitudinal polarization of the \( e^+e^- \) beams used, 520, 320, and 300 production events are possible in the processes \( e^+e^- \rightarrow hh \) (at \( M_h = 115 \) GeV), \( e^+e^- \rightarrow HH \), and \( e^+e^- \rightarrow AA \) (at \( M_H = 120 \) GeV), respectively. Even at \( M_{H,A} \approx 500 \) GeV and \( \sqrt{s} = 1.5 \) TeV, not less than 200 events for each of the processes can be accumulated. The cross section for the process \( e^+e^- \rightarrow hH \) is small (about \( 10^{-2} \) fb), which complicates the detection of the sought signal significantly.

PACS numbers: 12.38.Aw, 12.60.Jv, 14.80.Cp

DOI: 10.1134/S1063778809020197

1. INTRODUCTION

An experimental determination of Higgs boson coupling constants is one of the steps toward experimentally testing the Higgs mechanism within the Standard Model (SM) and within the minimal supersymmetric standard model (MSSM). In order to solve this problem, it is necessary first of all to analyze cross sections for processes determined by these constants in the leading approximation and to reveal, among others, processes that are characterized by maximum cross sections and by the highest sensitivity to variations in the coupling constants.

The main elementary processes for determining coupling constants at a \( e^+e^- \) linear collider are the following:

(I) \[ e^+e^- \rightarrow ZH_iH_j, \ ZAA; \]

(II) \[ e^+e^- \rightarrow \bar{\nu}_e\nu_eH_iH_j, \ \bar{\nu}_e\nu_eAA; \]

(III) \[ e^+e^- \rightarrow AH_iH_j, \ AAA; \]

(IV) \[ e^+e^- \rightarrow H_iH_j, \ AA, \ H_iA; \]

\( H_{i,j} = h, H. \)

The cross sections for processes (I) and (II), which are the most promising for determining the Higgs boson coupling constants, are 0.1 to 1 fb in the tree approximation. All processes in class (IV), with the exception of \( e^+e^- \rightarrow hh \), are characterized by extremely small cross sections (\( 10^{-6} \) to \( 10^{-9} \) fb) and cannot be recorded experimentally.

The smallness of the cross sections for these processes is due to the fact that all terms in their amplitudes are in direct proportion to the first or the second power of the Yukawa coupling constants. In turn, each such coupling constant involves the factor \( g_2 \frac{m_i}{M_W} \approx 6.4 \times 10^{-6} g_2 \). In view of the smallness of the ratio \( \frac{m_i}{M_W} \), the coupling constants in question are several orders of magnitude smaller than the weak coupling constant \( g_2 \), which characterizes the interaction of \( W^\pm \) and \( Z \) bosons with leptons.

In [1], the process \( e^+e^- \rightarrow HH \) was studied within the Standard Model in the one-loop approximation. That investigation was motivated by searches for processes dominated by loop corrections, which can be used as criteria for identifying a model. It turned out that, in models like the Standard Model, where the Lagrangian involves several coupling constants, there are special effects in perturbation theory—namely, specific one-loop contributions (those of one-loop diagrams not featuring Yukawa coupling constants) exceed considerably tree contributions...
and one-loop contributions not belonging to this type (those of tree diagrams and one-loop diagrams characterized by Yukawa coupling constants). For the example of the dependence $\sigma(M_H)$, it was demonstrated in [1] that the cross section for the process in question reaches values of $10^{-1}$ to 10 fb.

Later, this perturbation-theory effect was studied within the minimal supersymmetric standard model in [2]. For the example of the processes $e^+e^- \to hh$, $hh$, $HH$, $AA$, it was shown there that, in this case inclusive, their cross sections in a specific region of the parameter space of this model are on the same order of magnitude as their Standard Model analogs. The calculation was performed in the approximation of rigorous chiral symmetry ($m_e \to 0$). In calculating relevant amplitudes, the authors of [2] took into account only four-point one-loop diagrams. They disregarded the contributions of the remaining diagrams, relying on their vanishing in the limit $m_e \to 0$. In my opinion, however, the disregard in [2] of some terms in the amplitude for the processes considered there is not obviously justified. The problem of the choice of gauge in calculating observables and some fine details in numerical calculations were not clarified in [2].

In view of the aforesaid, the main objective of the present study is to calculate the cross sections for the processes $e^+e^- \to hh$, $hh$, $HH$, and $AA$ within the minimal supersymmetric standard model in the complete one-loop approximation and in the chiral-symmetry approximation ($m_e \to 0$). In doing this, we aim at removing the aforementioned drawbacks and at assessing (i) the contribution of those terms to the ultimate result that were discarded in previous studies, (ii) the sensitivity of the cross sections for the above processes to variations in the coupling constants, and (iii) the loop contributions from superpartner particles to the cross sections in question.

2. BASIC TOOLS OF THE CALCULATIONS

In order to reach the goal pursued here, use is made of the following tools of quantum-field-theory calculations:

(i) The method of quantum-field-theory perturbation theory implemented in terms of Feynman’s diagrammatic approach is a basic method for solving the problem at hand.

(ii) The kinematics of $2 \to 2$ processes in the $m_e = 0$ approximation was parametrized in the form considered in detail in [1, 2].

In the case where primary $e^+e^-$ beams are unpolarized, the differential cross section for the processes in question in terms of Mandelstam variables can be represented in the form

$$
\frac{d\sigma}{dt} = \frac{1}{64\pi s^2(1 + \delta_{h\ell h_m})} \sum_{s_\pm, s} |A_{[i \to f]}|^2, \quad (1)
$$

$$
\delta_{h\ell h_m} = \begin{cases} 1, & h_\ell \equiv h_m, \\ 0, & h_\ell \neq h_m. \end{cases}
$$

In expression (1), the squared moduli of the amplitudes are summed over spins of the $e^+e^-$ beams used. The total cross section is then given by

$$
\sigma_{\text{tot}} = \int_{t_{\min}}^{t_{\max}} \left[ \frac{d\sigma}{dt} \right] dt. \quad (2)
$$

The explicit form of $t_{\{\min, \max\}}$ is presented in [2]. In order to calculate the cross section for the process being considered, it is necessary to calculate first its amplitude in terms of the Mandelstam variables: $A_{[i \to f]} = A_{[i \to f]}(s, t, u)$.

(iii) The 't Hooft–Feynman gauge is used in loop calculations.

(iv) The recipes of tensor [3, 4] and dimensional [5, 6] reduction are used in evaluating analytic expressions corresponding to Feynman diagrams.

(v) The on-shell scheme [7] of the renormalization procedure is employed to derive finite physical results.

(vi) As a rule, the results of loop calculations are represented in the form of linear combinations of Veltman–Passarino scalar integrals [8, 9]. In a numerical analysis of the results, use is made of both traditional [8] and new algorithms [10–12] for evaluating such integrals.

3. CALCULATION OF AMPLITUDES FOR THE PROCESSES UNDER ANALYSIS

In calculating the amplitudes $A_{[i \to f]}$, we adopt the following line of reasoning:

(i) We consider that the $m_e = 0$ approximation leads to the elimination of all diagrams involving Yukawa interaction vertices.

(ii) We systematize diagrams contributing to the amplitude in terms of a minimum set of skeleton diagrams; each such diagram involves a vertex function, which must be calculated in the one-loop approximation.

(iii) The square of the modulus of the amplitude for a process must be expressed in terms of the Mandelstam variables.

The set of skeleton diagrams for the processes $e^+e^- \to hh$, $e^+e^- \to HH$, and $e^+e^- \to HH$ that