Abstract—The hierarchy of fermion masses and electroweak (EW) symmetry breaking without elementary Higgs is studied on the basis of strong gauge-field distributions governing the EW dynamics. The mechanism of symmetry breaking due to quark bilinear condensation is generalized to the case, when higher field correlators are present in the EW vacuum. Resulting wave functional yields several minima of quark bilinears, giving masses of three (or more) generations. Mixing is suggested to be due to kink solutions of the same wave functional. For a special form of this mixing (“coherent mixing”) a realistic hierarchy of masses and CKM coefficients is obtained and arguments in favor of the fourth generation are given. Possible important role of topological charges for $CP$-violating phases and small masses of the first generation is stressed.

1. INTRODUCTION

Despite spectacular success of the Standard Model (SM), the Higgs sector and the pattern of fermion masses and mixing remain mostly an unsolved issue, for the theoretical overview see [1]. In the present paper we suggest a framework which might shed some light on the origin of generations, the hierarchy of fermion masses, and the Higgs problem.

The topics mentioned above are related to several problems:

(i) Dynamical origin of Higgs sector and spontaneous symmetry breaking in the $SU(2) \times U(1)$ sector;

(ii) Fermion generations and hierarchy of fermion mass matrices;

(iii) Origin of $CP$ violation.

Possible solutions of the Higgs problem, different from the popular SUSY scenario, have been suggested by technicolor model [2] and by economical idea of top condensate model [3–5]. There a strong interaction at high scale $M \sim 10^{15} – 10^{16}$ GeV allows to create Higgs sector dynamically, but leaves points (ii) and (iii) unsolved.

A way to understand large top mass was suggested already 30 years ago [7] and developed in detailed manner since then [8, 9]. The symmetry responsible for large top mass was called “flavor democracy” and considered in family space in each of the sectors (up, down, and leptons) separately. The realization of this symmetry in the framework of the “flavor gauge theory” was given in [10].

The flavor-democratic scenario illustrates why in each of the sectors the mass of the third family is much larger than that in first two families and allows to connect phenomenologically the CKM mixing angles with masses [11]. However, it does not consider dynamical origin of first two families and another important hierarchy: why scales of the masses in three sectors are so much different, and inside the family the mass of top is much larger, than that of bottom and tau-lepton.

Summarizing, the problem of lower generations was not addressed. It is remarkable that masses of the first generation have much smaller scale, which might signify that internal dynamics may differ from generation to generation. Also the dynamical mechanism producing generations remains unknown. It is a purpose of the present paper to suggest a possible variant of such mechanism, based on nonperturbative dynamics of the electroweak (EW) gauge and fermion fields. We will show below that the fermion masses due to Spontaneous Symmetry Breaking (SSB) naturally form the family structure, when higher field correlators are taken into account.

We also show that dilute topological charges in the EW vacuum may be responsible for the dynamics of the lowest fermion family. This formalism can be used for producing Higgs phenomenon in the same way as it was done in the topcolor-type models [3–6].

In this way the Higgs is coupled to (and made of) all fermions, and the scalar condensate is formed
dynamically, giving mass to all quarks. The field-theoretical framework allows to consider additional contributions from topological charges creating nonzero masses for light fermions of first generation. The fermion mixing is associated with the kink solutions of the same wave functional, which connect different stationary points corresponding to generations. For a special form of the mass matrix, called the coherent mixing form, the mass eigenvalues have a pronounced hierarchy and CKM mixing coefficients are expressed via the mass ratios yielding realistic values. The neutrino mass can be considered on the same ground, including leptons and quarks symmetrically, and then the mixing, both in quark and lepton sectors, is obtained in the same way.

The plan of the paper is as follows. In Section 2 the gauge interaction at intermediate scale is introduced and resulting multifermion Lagrangian is derived. The gap equation is solved and the mass matrix is obtained and discussed in Section 3. Contributions of topological charges are given in Section 4. The problem of the fermion mixing is studied in Section 5, while the SSB and Higgs dynamics is presented in Section 6. Section 7 is devoted to summary and possible developments of the method. Three Appendices contain an additional material for derivation of formulas in the text: Appendix A yields the quark Green’s function in the field of topological charges; Appendices B and C describe diagonalization of the mass matrices in the case of three and four generations.

2. DERIVATION OF THE MULTIFERMION LAGRANGIAN

The SM Lagrangian can be split in two parts,

\[ L_{\text{SM}} = L_{\text{st}} + L_{\text{Higgs}}, \]

where \( L_{\text{st}} \) contains all kinetic parts of fermions and gauge bosons and their interaction, whereas \( L_{\text{Higgs}} \) refers to all terms where the Higgs field appears. It is our purpose, as in [3–5], to derive \( L_{\text{Higgs}} \) with effective Higgs field from the fields present in \( L_{\text{st}} \), which would generate dynamically Higgs condensate, fermion masses, and mixings.

To this end, first of all, we must organize fermions into some structures which enter the fundamental Lagrangian, namely,

\[ \psi \in \{ \psi_{Li}^{\alpha}, \psi_{Ri}^{\alpha} \}, \quad A = \{ n, \alpha \}, \]

where \( \alpha = 1, 2, 3 \) refers to families and \( n = 1, 2, 3, 4 \) refers to “sectors” of fermions, which can be composed as follows:

\[ \psi_{Li}^{1,\alpha} = (e_{L}^{\alpha}, \mu_{L}^{\alpha}, \tau_{L}^{\alpha}), \]

\[ \psi_{Li}^{2,\alpha} = (\nu_{eL}^{\alpha}, \nu_{\mu L}^{\alpha}, \nu_{\tau L}^{\alpha}), \]

\[ \psi_{Ri}^{3,\alpha} = (u_{R}^{\alpha}, c_{R}^{\alpha}, t_{R}^{\alpha}), \]

\[ \psi_{Ri}^{4,\alpha} = (d_{R}^{\alpha}, s_{R}^{\alpha}, b_{R}^{\alpha}), \]

\[ \psi_{Ri}^{5,\alpha} = (e_{R}^{\alpha}, c_{R}^{\alpha}, t_{R}^{\alpha}). \]

Note that considering the gauge dynamics at high scale \( M \), one can introduce, similarly to [11], the “urfermions” with quantum numbers which are possibly different from those of final diagonalized fermions in (2)–(5).

Urfermions are denoted by the hat sign, \( \psi_{a}^{\hat{A}} \), and supplied by an additional index \( a \), implying that \( \psi_{a}^{\hat{A}} \) belongs to a representation of some gauge group \( G \) operating at the scale \( M \). We shall assume here that this group is broken at low-scale and only one, the lowest mass component \( a = 1 \), should be considered for low-scale dynamics. The diagonalized form of \( \psi_{a=1}^{\hat{A}} \) will be associated with the physical states listed in (2)–(5).

The destiny of higher states, \( \psi_{a}, a = 2, 3, \ldots \), will be discussed elsewhere, together with a possibility that the sets \( \{ a \} \) and \( \{ A \} \) have a common intersection. In what follows we consider the simplest case with standard fermions listed in sectors (2)–(5).

The fundamental Lagrangian at high scale reads (the Euclidean fields and metrics are used everywhere)

\[ L_{\text{high}} = g \psi_{a}^{A} (x) \gamma_{\mu} C^{ab}_{\mu}(x) \psi_{b}^{A}(x), \]

\[ C^{ab}_{\mu} = C^{ab}_{\mu} T_{ab}^{a}. \]

The generating functional can be written as

\[ Z = \text{const} \cdot \int D\mu(C) D\hat{\psi} D\hat{\bar{\psi}} \]

\[ \times \exp \left( i \int \hat{\bar{\psi}} \hat{\psi} d^{4}x + \int d^{4}x L_{\text{high}} \right), \]

where \( D\mu(C) \) is the integration over gauge field \( C_{\mu} \) with the standard weight, which may be also considered as the averaging over vacuum fields \( C_{\mu} \), denoted...