Test of the Glauber Formula for Nucleon–Deuteron Scattering at Intermediate Energies

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Abstract—A high-precision test of the Glauber formula for the amplitude of nucleon–deuteron scattering is performed. Nucleon–nucleon amplitudes used in the calculations depend on the spins of interacting particles, phase shifts, and mixing parameters. These amplitudes were derived by using the Nijm I, Nijm II, Njim 93, and Reid 93 realistic potentials. The differential cross sections for nucleon–deuteron scattering were calculated for the projectile-nucleon energies of 65, 95, 135, 150, 190, and 250 MeV, and the results of these calculations were compared with experimental data.

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Investigation of collisions between high-energy particles and nuclei is an important problem within which one can study nucleon-density distributions, the dynamics of nucleon–cluster formation in nuclei, the color properties of quark structures, and so on. From the microscopic point of view, a consistent description of quantities observed in these reactions is an extremely difficult theoretical problem [1]. In view of this, the number of currently existing successful theoretical approaches is relatively small here, the Glauber method being among them [2, 3]. In [1], it was indicated, with good reason, that, although this important method has extensively been used in various scattering problems for more than half a century, only very few studies, surprisingly as it is, have been devoted to rigorously verifying the accuracy and the applicability range of the Glauber ansatz. It is the opinion of the present author that, for a first step along these lines, we could address the problem of a more rigorous derivation of form for the nucleon–nucleon amplitude. This amplitude has the most general form and depends on the spins of colliding particles, their energy, phase shifts, and mixing parameters. Using it in the well-known formula of the eikonal approximation for the amplitude of nucleon scattering on a deuteron [3], we can directly test the correctness of the Glauber approach for the case being considered.

The differential cross section for nucleon–deuteron scattering is calculated here by the formula [5] (below, use is everywhere made of the c.m. frame and the system of units where $h = c = 1$)

$$\sigma(\theta) \equiv \frac{d\sigma}{d\Omega} = \frac{1}{6} \text{tr} \left( F_d F_d^\dagger \right),$$

where $F_d$ is the amplitude for nucleon–deuteron scattering. It has the form [3]

$$F_d(q) = \frac{k_d}{k_1} F_1(q) G \left( \frac{q}{2} \right) + \frac{k_d}{k_2} F_2(q) G \left( \frac{q}{2} \right)$$

$$+ \frac{ik_d}{2\pi k_1 k_2} \int d(2) g F_1 \left( g + \frac{q}{2} \right) F_2 \left( g - \frac{q}{2} \right) G(g),$$

$$G(g) = \int dr \phi_d^2(r) \exp(i g \cdot r), \quad q \cdot g \perp k_d,$$

where $k_d$ is the deuteron momentum, $k_1$ and $k_2$ are the momenta of the deuteron nucleons, $F_{1,2}$ are the amplitudes for projectile-nucleon scattering on the deuteron nucleons, $q$ is the momentum transfer, and $\phi_d(r)$ is the ground-state deuteron wave function.

The sum of the first two terms in expression (2) is the scattering amplitude in the impulse approximation, $F_d^{(i)}$ (it corresponds to taking into account single collisions between the projectile nucleon and the deuteron nucleons). The last term in expression (2) is the so-called shadowing correction $F_d^{(sh)}$, it corresponds to the contribution of double scattering to the amplitude, $F_d^{(d)}$.

$$F_d = F_d^{(i)} + F_d^{(sh)}.$$
With allowance for the change invariance of the nucleon–nucleon interaction, the amplitudes $F_j$ ($j = 1, 2$) have the form [4, 6]

$$F_j = f_{1j} + f_{2j}(n\sigma_j)(n\sigma) + if_{3j}n(\sigma_j + \sigma)$$

$$+ f_{4j}(m\sigma_j)(m\sigma) + f_{5j}(l\sigma_j)(l\sigma),$$

where $f_{sj}$ ($s = 1, 2, \ldots, 5$) are coefficients that depend on the energy of colliding nucleons, phase shifts, and mixing parameters (see Appendix A); $\sigma$ is the projectile–nucleon spin operator; and $n$, $m$, and $l$ are three mutually orthogonal unit vectors defined as

$$n = \frac{k_j \times k'_j}{|k_j \times k'_j|}, \quad m = \frac{k_j - k'_j}{|k_j - k'_j|}, \quad l = \frac{k_j + k'_j}{|k_j + k'_j|},$$

where $k_j$ ($k'_j$) is the momentum of the incident (scattered) $j$th nucleon ($k_j = k_d/2 = k$).

Evaluation of the trace in Eq. (1) with allowance for Eqs. (2) and (5) leads to the expression

$$\text{tr} \left( F_d F_d^\dagger \right) = 4 \left| G \left( \frac{q}{2k} \right) \right|^2 S_{11}$$

$$+ \frac{2}{\pi k} \left( \frac{q}{2k} \right) S_{12} + \left( \frac{1}{\pi k} \right)^2 S_{22},$$

where $S_{11}$, $S_{12}$, and $S_{22}$ are quantities that depend on the coefficients appearing in the amplitudes $f_j$ (see Appendix B). The calculations of the differential cross sections in (1) were performed for the projectile–nucleon energies of 65, 95, 135, 150, 190, and 250 MeV by using the nucleon–nucleon phase shifts, mixing parameters, and deuteron wave functions obtained with the Njim I, Njim II, Njim 93, and Reid 93 potentials [7]. Here, we disregarded relativistic corrections, since, in the energy range being considered, their effect on the cross section is insignificant (see [1] and references therein).

An analysis of the calculated cross sections in the Fig. 1 leads to the following conclusions:

(i) For a given projectile–nucleon energy $E_N$, the relative contribution of double scattering, $F_d^{(sh)}$, is virtually independent of the angle $\theta$ and decreases as $E_N$ grows. In the Glauber approximation (the deuteron size exceeds considerably the range of the nucleon–nucleon interaction), the shadowing correction decreases more slowly in relation to $F_d^{(i)}$ as the angle $\theta$ grows [18, 19].

(ii) It is traditionally assumed [2, 20] that the Glauber formula (2) is valid for $q \ll k$ and $k^{-1} \ll R_{\text{rms}}$, where $R_{\text{rms}}$ is the root-mean-square radius of the deuteron. This condition is independent of energy and yields the following estimate for the angle $\theta$: $\theta < 10^\circ$. In fact, the Glauber formalism works well even beyond the original assumptions of the theory (see, for example, [21]). The precise calculations performed in the present study for the aforementioned cross sections with the above realistic potentials only confirm this well-known fact: the calculated curves describe experimental data in the angular range $\theta < 30^\circ$ and in the energy range 65–150 MeV; as the projectile–nucleon energy increases further, the angular range becomes broader: $\theta < 70^\circ$ (for 190 and 250 MeV).

Thus, the test performed here for the Glauber formula in the case of nucleon–deuteron scattering by using realistic nucleon–nucleon potentials gives sufficient ground to conclude that, in the range of projectile–nucleon energies $E_N$ that is considered here, the theory in question works well beyond the constraint $q \ll k$. Moreover, the angular range broadens to $\theta < 70^\circ$ as soon as the quantity $k^{-1}$ becomes commensurate with $R_{\text{rms}}$, not only much less than it.