Low-Energy Parameters of Neutron–Neutron Interaction in the Effective-Range Approximation

V. A. Babenko and N. M. Petrov
Bogolyubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine, Metrologicheskaya ul. 14б, 03143 Kyiv, Ukraine
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Abstract—The effect of the mass difference between the charged and neutral pions on the low-energy parameters of nucleon–nucleon interaction in the $^1S_0$ state is studied in the effective-range approximation. On the basis of experimental values of the singlet parameters of neutron–proton scattering and the experimental value of the virtual-state energy for the neutron–neutron system in the $^1S_0$ state, the following values were obtained for the neutron–neutron scattering length and effective range: $a_{nn} = -16.59(117)$ fm and $r_{nn} = 2.83(11)$ fm. The calculated values agree well with present-day experimental results.

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1. INTRODUCTION

Low-energy parameters of nucleon–nucleon ($NN$) interaction are fundamental quantities that play a significant role in studying strong nucleon–nucleon interaction. These quantities are also of importance in constructing various realistic nuclear-force models, which, in turn, form a basis for exploring the structure of nuclei and various nuclear processes.

Investigations of low-energy parameters of nucleon–nucleon interaction in the spin-singlet ($^1S_0$) state are of particular importance in connection with the problem of testing the hypothesis of charge independence and charge symmetry of nuclear forces. This hypothesis is confirmed to some extent by an approximate equality of binding energies of isobar nuclei. However, experimental and theoretical investigations aimed at quite a precise determination of low-energy parameters for neutron–proton ($np$), proton–proton ($pp$), and neutron–neutron ($nn$) interactions are of paramount importance for definitively solving this problem.

The parameters of neutron–proton and proton–proton systems can be determined to a high degree of precision directly from experiments. At the same time, it is impossible to study neutron–neutron scattering directly because of the absence of neutron targets. In order to determine experimentally the parameters of neutron–neutron interaction, use is usually made of nuclear reactions leading to the production of two interacting neutrons in the final state. A detailed review of experimental and theoretical methods for determining the neutron–neutron scattering length $a_{nn}$ from data on such reactions was given in [1].

On the basis of a vast body of experimental data obtained before 1974, a mean-weighted value of $a_{nn} = -16.61(54)$ fm was found in [1] for the neutron–neutron scattering length. It is noteworthy that this value is consistent with present-day experimental results.

The deuteron-breakup reaction $n + d \rightarrow p + n + n$ is one of the reactions leading to the appearance of two interacting neutrons in the final state. A fit to data on this reaction on the basis of the Migdal–Watson formula [2, 3] makes it possible to determine the value of the virtual-state energy $\varepsilon_{nn}$ for the neutron–neutron system in the $^1S_0$ state [4–6].

At the present time, a description of nucleon–nucleon interaction frequently relies on semiphenomenological one-boson-exchange potential models involving the exchange of various mesons. Within this approach, the exchange of pions, which are light, determines primarily the long-range part of the nucleon–nucleon potential, while the exchange of heavier rho and omega mesons determines the interaction at intermediate and short distances, which is significant at higher energies. At extremely low energies, which effectively correspond to long distances, the use of extremely simple one-pion-exchange potentials in describing nucleon–nucleon interaction is quite reasonable. Taking into account the foregoing and assuming that nuclear forces in the nucleon–nucleon system at low energies are due primarily to the exchange of virtual pions, we study here, in the effective-range approximation, the influence of the mass difference between the charged and neutral pions on low-energy parameters of nucleon–nucleon scattering in the $^1S_0$ state. We
find a relation between low-energy parameters of the neutron–proton system and their counterparts for the neutron–neutron system. The use of this relation and the energy of a virtual level in the neutron–neutron system makes it possible to determine the scattering length \(a_{nn}\) and the effective range \(r_{nn}\).

2. BASIC RELATIONS OF THE EFFECTIVE-RANGE APPROXIMATION

It is important to investigate the low-energy parameters of nucleon–nucleon interaction in the spin-singlet \((1S_0)\) state in view of the need for testing the hypothesis of charge independence and charge symmetry of nuclear forces. The effective-range approximation\[7, 8\]

\[
k \cot \delta_{NN} = -\frac{1}{a_{NN}} + \frac{1}{2} r_{NN} k^2,
\]

where \(a_{NN}\) and \(r_{NN}\) are, respectively, the scattering length and the effective range for nucleon–nucleon interaction, is the most convenient and the most popular method for analyzing experimental data on nucleon–nucleon scattering at low energies. If use is made of the approximation specified by Eq. (1), the \(S\) matrix can be written in the form

\[
S(k) = \frac{(k + i\alpha_{NN})(k + i\beta_{NN})}{(k - i\alpha_{NN})(k - i\beta_{NN})},
\]

where

\[
\alpha_{NN} = \frac{1}{r_{NN}} \left[ 1 - \left( 1 - \frac{2r_{NN}}{a_{NN}} \right)^{1/2} \right],
\]

\[
\beta_{NN} = \frac{1}{r_{NN}} \left[ 1 + \left( 1 - \frac{2r_{NN}}{a_{NN}} \right)^{1/2} \right].
\]

The \(S\) matrix in (2) has two poles in the complex plane of the wave number \(k\). The pole \(i\alpha_{NN}\) situated in the lower half-plane of \(k\) corresponds to a virtual \((\alpha_{NN} < 0)\) state of the two-nucleon system at the energy

\[
\epsilon_{NN} = \frac{\hbar^2 \alpha_{NN}^2}{2m_{NN}},
\]

where \(m_{NN}\) is the reduced mass of the two-nucleon system. The second pole \(i\beta_{NN}\) \((\beta_{NN} > 0)\) situated in the upper half-plane of \(k\), is the well-known redundant pole of the \(S\) matrix.

The potential corresponding to the effective-range approximation (1) has the form\[9–11\]

\[
V_{NN}(r) = -V_{NN0} \frac{e^{-r/R_{NN}}}{\left( 1 + \gamma_{NN} e^{-r/R_{NN}} \right)^2},
\]

where

\[
R_{NN} = \frac{1}{2\beta_{NN}},
\]

\[
V_{NN0} = \frac{\hbar^2}{m_{NN} R_{NN}^2} \gamma_{NN},
\]

\[
\gamma_{NN} = 1 + 2\alpha_{NN} R_{NN} \frac{1 - 2\alpha_{NN} R_{NN}}{1 - 2\alpha_{NN} R_{NN}}.
\]

At large distances, the potential in (6) decreases exponentially; that is,

\[
V_{NN}(r) \simeq V_{NN0} e^{-r/R_{NN}}.
\]

As can be seen from Eq. (7), the potential range \(R_{NN}\) is determined directly in terms of the parameter \(\beta_{NN}\), which characterizes the redundant pole of the \(S\) matrix in (2). In the case of the approximation specified by Eq. (1), Eqs. (3) and (4) can be used together with Eq. (7) to derive equations relating low-energy parameters of the nucleon–nucleon system to the interaction range \(R_{NN}\)

\[
r_{NN} = 4R_{NN} \left( 1 - \frac{2R_{NN}}{a_{NN}} \right),
\]

\[
r_{NN} = 4R_{NN} / (1 + 2\alpha_{NN} R_{NN}).
\]

Relations (11) and (12), together with the values of the interaction range \(R_{NN}\) and virtual-state energy (5), make it possible to determine the nucleon–nucleon scattering length \(a_{NN}\) and effective range \(r_{NN}\).

3. DETERMINATION OF LOW-ENERGY PARAMETERS OF PROTON–PROTON AND NEUTRON–NEUTRON SCATTERING

A comparison of the scattering lengths and effective ranges for neutron–proton, proton–proton, and neutron–neutron scattering is one of the methods that makes it possible to verify the hypotheses of charge independence and charge symmetry of nuclear forces. In this case, one can also compare the values of the virtual-state energies in the aforementioned nucleon–nucleon systems. In comparing low-energy parameters of nucleon–nucleon interaction, one should bear in mind that, in the case of proton–proton interaction, it is necessary to use purely nuclear low-energy parameters obtained by excluding the electromagnetic component of the proton–proton interaction from the experimental values of the parameters in question.

According to Yukawa’s meson theory, strong nuclear interaction between two nucleons is due largely to the exchange of virtual pions, which determines the long-range part of the nucleon–nucleon interaction.