Form Factors with $q^2 = 0$ and Grassmannians in $\mathcal{N} = 4$ Sym Theory

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Abstract—In this note we consider tree level form factors of operators from stress tensor supermultiplet with light like operator momentum $q^2 = 0$. The presentation of form factors in terms of the regulated integral over Grassmannian is given. The conjectured formula is verified by successfully reproducing known answers in the MHV and $N^{k-2}$MHV, $k \geq 3$ sectors as well as appropriate soft limit behavior.

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INTRODUCTION

A recent progress in understanding the structure of $S$-matrix (amplitudes) of $\mathcal{N} = 4$ SYM theory and other four dimensional gauge theories (for review see [1] and reference therein) is based on the development of new computational techniques and ideas. One of such new ideas is the discovery of the representation of the elements of tree level $S$-matrix of $\mathcal{N} = 4$ SYM theory and leading singularities of loop amplitudes in terms of the integrals over Grassmannians [2, 3].

Another interesting objects to study are given by form factors. The latter are defined as matrix elements of local gauge invariant operators $O$ between vacuum and on-shell states $\langle p_1^{\lambda_1}, ..., p_n^{\lambda_n} | O | 0 \rangle$ with momenta $p_1, ..., p_n$ and helicities $\lambda_1, ..., \lambda_n$. That is,

$$ F^C_{\mathfrak{n}} = \langle p_1^{\lambda_1}, ..., p_n^{\lambda_n} | [O] | 0 \rangle. $$

The form factors in $\mathcal{N} = 4$ SYM was initially considered in [4], almost 20 years ago. After nearly a decade the investigation of 1/2-BPS form factors was again undertaken in [5–7] and now it is an active research topic, see [15, 16] and references therein for latest developments.

In this letter we continue the study of Grassmanian representation for form factors initiated in [17, 18] and present a new Grassmanian integral representation of form factors of operators from $\mathcal{N} = 4$ SYM stress tensor supermultiplet for the case of light like momentum $q^2 = 0$ of operator. Being the most simple this case nevertheless captures all the essential differences of form factors compared to amplitudes.

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imal” form factor $Z_n^{(k)}$ could be written as integral over the “small” Grassmannian of the form:

$$Z^{(2)}_2(1, 2) = \int d\alpha_1 d\alpha_2 \text{Reg}(1, 2 | q) \delta^2 (\tilde{\alpha}_1 + \alpha_1 \tilde{\alpha}_2)$$

$$\times \delta^4 (\eta_1 + \alpha_1 \eta_3) \delta^4 (\eta_2 + \alpha_2 \eta_3),$$

where the following notation for the inverse soft factor $S^{-1}$ was used:

$$\text{Reg}(i, i + 1 | q) = S^{-1}(i, q, i + 1) = \frac{\langle iq \rangle \langle qi + 1 \rangle}{\langle i + 1 \rangle}.$$  

This factor could be further written as a ratio of some none consecutive minors $M$ and $M'$ of $C_{ai}$ matrix: $\text{Reg} = \langle iq \rangle M / M'$. This observation allows us to suggest, that the appropriate deformation (regularization of the soft limit behavior) Grassmannian representation for amplitudes to the case of form factors could be archived with the insertion of $\text{Reg}$ functions written in the form of the anzats

$$\text{Reg} = \sum_i \langle iq \rangle \frac{M_{a(i)}^{(i)}}{M_{b(i)}^{(i)}},$$

where $M_{a(i)}^{(i)}$ and $M_{b(i)}^{(i)}$ are some (in general non-consecutive) minors of $C_{ai}$ matrix. Next, using the results of BCFW recursion for $\text{MHV}_n$

$$Z^{(2)}_n = \langle ql \rangle \frac{(nn + 13 \ldots k)}{(ln3 \ldots k)},$$

and $N^{k-2}\text{MHV}_{k+1}$ form factors (green arrow in Fig. 1)

$$Z^{(k)}_{k+1} = \sum_{j=4}^{k+1} \langle q | p_i + p_j + \ldots + p_{k+1} | 2 \rangle$$

$$\times A^{(k)}_{k+2}(1, \ldots, j - 1, q, j, \ldots, k + 1)$$

$$+ \frac{\langle q | p_2 | 2 \rangle}{[q2]} A^{(k)}_{k+2}(1, \ldots, k + 1, q),$$

we can fix the explicit form of $M_{a(i)}^{(i)}$ and $M_{b(i)}^{(i)}$ and write a conjecture for the Grassmannian representation for form factors of operators from stress tensor supermultiplet at $q^2 = 0$:

$$Z^{(k)}_n = \sum_{j=1}^{k+1} \int d^{n+1|k} C_{ai} \text{Reg}^{R(k)}_j$$

$$\times \delta^{44}(1, \ldots, j - 1, q, j, \ldots, n)$$

$$+ \int \text{Vol}(GL(k)) \text{Reg}^{L(k)}_n \delta^{44}(1, \ldots, n, q),$$

where $\text{Reg}^{R(k)}_j$ and $\text{Reg}^{L(k)}_n$ are given by

$$\text{Reg}^{R(k)}_j = \langle ql \rangle \frac{(k + 1 + 2 \ldots j)}{(13 \ldots j - 1 + 1 \ldots n + 1)},$$

$$\text{Reg}^{L(k)}_n = \langle ql \rangle \frac{(nn + 13 \ldots k)}{(ln3 \ldots k)},$$

for $k \geq 3$ and

$$\text{Reg}^{R(2)}_j = 0, \quad \text{Reg}^{L(2)}_n = \langle ql \rangle \frac{(nn + 1 + 13 \ldots k)}{(ln3 \ldots k)};$$

for $k = 2$. Here we introduced the following short notation for the combination of delta functions in the Grassmannian integral

$$\delta^{44}(1, \ldots, l, q, i + 1, \ldots, n)$$

$$= \prod_{a=1}^{k} \delta^{2} \left( \sum_{l=1}^{n+1} C_{a \tilde{\lambda}_l} \right) \delta^{4} \left( \sum_{l=1}^{n+1} C_{\tilde{\lambda}_l} \right) \prod_{b=k+1}^{n+1} \delta^{2} \left( \sum_{l=1}^{n+1} \tilde{C}_{a \tilde{\lambda}_l} \right).$$

To verify our conjecture we checked that, for example, $N^{3}\text{MHV}_5$, $N^{3}\text{MHV}_6$ and $N\text{MHV}_5$ form factors are reproduced correctly. In addition we verified that the conjectured Grassmannian integral representation correctly reproduces soft behavior with respect to $q \mapsto 0$ limit:

$$Z^{(k)}_n |_{q \mapsto \infty} = (k - 1)A^{(k)}_n(1, \ldots, n) + \mathcal{O}(\epsilon).$$