Zero-Sound Condensate in a Fermi Liquid1, 2

E. E. Kolomeitseva, * and D. N. Voskresenskyyb

aMatej Bel University, Banska Bystrica, SK-97401 Slovakia
bNational Research Nuclear University “MEPhI”, Moscow, 115409 Russia
*e-mail: E.Kolomeitsev@gsi.de

Abstract—We consider a normal Fermi liquid with a local scalar interaction given by the Landau parameter $f_0$. The system becomes unstable for $f_0 < -1$ against a growth of scalar-mode excitations (Pomeranchuk instability). We show that the instability may be tamed by the formation of a static Bose condensate of the scalar modes. We study a possible reconstruction of the isospin-symmetric nuclear matter owing to the appearance of the condensate. Possibility of a novel metastable state at subnuclear densities is demonstrated.

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1. INTRODUCTION

Consider a normal Fermi liquid of non-relativistic fermions stable against pairing [1]. The particle–hole (p–h) scattering amplitude on the Fermi surface is determined by the infinite series of p–h diagrams dressing the fermion Green’s function, $N = \nu m^* p_F/\pi^2$ is the density of states at the Fermi surface, $m^*$ is the effective fermion mass. The Fermi momentum $p_F$ is related to the total fermion density as $n = \nu p_F^3/(3\pi^2)$ with $\nu = 1$ for one type of fermions and $\nu = 2$ for two types of fermions, like for the isospin-symmetric nuclear matter. The Landau parameters $f_i$ can be calculated or fitted from experiments. The scalar parameter $f_0$ is related to the incompressibility of the system $K_t = n a^2 E_t/\partial n^2 = (1 + f_0) p_F^2/3m^*$, where $E_t$ is the energy density of the fermion system. The p–h scattering amplitude induced by the $f_0$ parameter is [1]

$$T_{p,h}(\omega, k) = \left[a^2 N(f_0^{-1} + \Phi(\omega, k))\right]^{-1}, \quad \Phi(\omega, k) = \frac{1}{2} + \frac{\sum_{i=1}^{n} (-i) \frac{z_i^2 - 1}{4(z_i - 1)} \ln \frac{z_i + 1}{z_i - 1}}{\omega + k - 2p_F},$$

is the Lindhard function, $\omega k$ are the energy and momentum transferred in the p–h channel.

2. SPECTRUM OF SCALAR EXCITATIONS

Solution of equation $f_0^{-1} = -\Phi(\omega, k)$ gives the spectrum of excitations in the scalar channel $\omega(k)$. For the repulsive interaction $f_0 > 0$ there exists a real solution of this equation $\omega = k v_F s(k/p_F)$, $s(k/p_F) = s_0 + s_2(k/p_F)^2$, $k/p_F \ll 1$, $v_F = p_F/m^*$ is the Fermi velocity. This solution is called the zero sound. The zero sound mode exists as a quasi-particle mode for frequencies much larger than the fermion collision time, i.e. $\omega \gg \epsilon_F T^2$, $\epsilon_F$ is the Fermi energy, $T$ is the temperature. In the opposite limit the solution describes a hydrodynamic (first) sound. For $k > k_{lim} \ll p_F$ the spectrum branch enters the region with $3\Phi > 0$, and the zero sound becomes damped.

If $f_0 < 0$ the equation has purely imaginary solution which for $k \ll p_F$ is $\omega(k) = i(2k v_F/\pi) \times \left[z_f - k^2/12p_F^2\right]$, $z_f = 1 - 1/|f_0|$. For $f_0 < -1$, $z_f > 0$ and in some interval of momenta $3\Phi(k) > 0$. The mode becomes exponentially growing (Pomeranchuk instability). Usually this instability is treated as a spinodal instability resulting in the creation of aerosol-like mixture of droplets and bubbles. We propose a new alternative that at certain conditions the Pomeranchuk instability may lead to formation of a static Bose condensate of a scalar field.
3. EFFECTIVE LAGRANGIAN OF SCALAR FIELD

In a fermion system with a contact interaction in scalar channel one can introduce a collective scalar bosonic field by means of the Hubbard–Stratonovich transformation [2], or by formal replacement of the contact interaction to exchange by a heavy scalar boson [3]. The effective Lagrangian of the static scalar condensate field taken in the simplest form $\varphi = \varphi_{0}e^{\imath k_{F}r}$ can be presented as follows

$$\mathcal{L}_{\varphi} = -\text{sgn}(f_{0})\left[\left(\Gamma_{0}^{\mu}\right)^{-1} + a^{2}N\Phi(0,k_e)\right] \times |\varphi_{0}|^2 - \Lambda(0,k_e)|\varphi_{0}|^4,$$

where the self-interacting term is determined by the integral of four fermion Green’s functions evaluated in [4] as $\Lambda(0,k_e) = a^4\lambda\left(1 + k_e^2/2p_{F}^2\right)$ for $k_e \ll p_{F}$ with $\lambda = \sqrt{(\pi^2v_{F}^3)}$. Variation of the Lagrangian in $\varphi_{0}$ yields equation $-a^2N\tilde{\omega}(k_e)\varphi_{0} - \Lambda(0,k_e)|\varphi_{0}|^2\varphi_{0} = 0$, $\tilde{\omega}(k_e) = |f_{0}(k_e)|^{-1} - \Re\Phi(0,k_e)$ is an effective gap in the excitation spectrum. The amplitude of the condensate field $|\varphi_{0}|$ and the Bose condensate energy density term become $||\varphi_{0}|^2 = -N\tilde{\omega}(k_e)\left(1 + k_e^2/2p_{F}^2\right)^{-1}$, $E_{b} = -N^2\tilde{\omega}(k_e)\left(1 + k_e^2/2p_{F}^2\right)^{-2}$. In the presence of the condensate the incompressibility becomes, $K = K_{f} + K_{b}$, $K_{b} = n\frac{dE_{b}}{dc}$, and, therefore, the scalar Landau parameter changes

$$f_{0} \rightarrow f_{0}^{\text{tot}} = f_{0} + f_{0}^{b} = f_{0} + 3m^{*}K_{b}\left[f_{0}^{\text{tot}}\right]/p_{F}^2.$$ (2)

Here $f_{0}$ and $f_{0}^{b}$ are functions of $k_e$ which is to be found from the energy minimization. In case of a weak condensate (for $|f_{0}| \ll 1$) one can use $E_{b}[/f_{0}^{\text{tot}}] = E_{b}[f_{0}]$. For a developed condensate the perturbative analysis does not work and one should solve Eq. (2) self-consistently.

4. STABILITY OF THE SYSTEM WITH CONDENSATE

Let us now study excitations on top of the condensate. Their spectrum is described by the equation

$$a^{2}N|\tilde{f}_{0}^{\text{tot}}(k)|^{2} + \Phi(\alpha,k)\delta\Sigma_{\varphi} = 0,$$

where $\delta\Sigma_{\varphi} = 2\Lambda(\alpha k)|\varphi_{0}|^2$ includes the interaction of excitations with the condensate. Making use expression for $|\varphi_{0}|$, we can cast the spectrum in the form $\omega = \imath\frac{2}{\pi}\tilde{\omega}(k_e)k_{F}v_{F}$ for $-1 \ll \tilde{\omega}(k_e) < 0$. We see that the excitations are damped: in the presence of the condensate the Fermi liquid is free from the Pomeranchuk instability of the zero-sound-like modes.

The p–h interaction also changes in the presence of the condensate. There appears a new term in equation for the p–h amplitude, which can be included in the renormalized local interaction as $1/f_{\text{ren,0}} = 1/f_{\text{tot}}^{(0)}(k_e) + 2\tilde{\omega}(k_e)$. For homogenous condensate with $k_e = 0$ we have $\tilde{\omega}(0) = -\epsilon_{f} = -1 - 1/f_{0}^{\text{tot}}$ and consequently $f_{\text{ren,0}} = -f_{0}^{\text{tot}}/(2f_{0}^{\text{tot}} + 1)$. Thus if originally $f_{0} < -1$ and therefore $f_{0}^{\text{tot}} < -1$, the renormalized interaction $-1 < f_{\text{ren,0}} < -1/2$. Hence, in the Fermi liquid with the condensate the first sound modes are stable. Knowing the value $f_{\text{ren,0}}$ one can reconstruct the energy density $E_{\text{tot}}(n)$ of the Fermi liquid from the differential equation

$$d^{2}E_{\text{tot}}(n)/dn^{2} = 2\epsilon_{F}(1 + f_{\text{ren,0}})/(3n),$$ (3)

which solution should continuously match the original energy-density $E_{f}$ at the values of the density where $f_{0} = -1$. Note that this energy density includes both mean-field and quadratic-fluctuation contributions.

Several approximations are done in our study. In $\mathcal{L}_{\varphi}$ we kept only terms up to $|\varphi|^4$. We disregarded self-interaction of excitations on top of the condensate and neglected feedback of fluctuations on the mean field. Thus, our results are valid if on the one hand $\varphi_{0}$ is rather small and on the other hand fluctuations on the top of the condensate yield a yet smaller contribution.

Let us demonstrate how the scheme works on example of the isospin-symmetric nuclear matter. We consider a system of 125 nucleons which energy per particle, $\tilde{\epsilon}_{N} = E_{N}/n - m_{N}$, contains the volume and surface parts. The volume part satisfies standard properties of the nuclear saturation: the density $n_{0} = 0.16$ fm$^{-3}$, the energy per particle $\tilde{\epsilon}_{0} = -16$ MeV and the nuclear incompressibility $9K_{f} = 285$ MeV. The inclusion of the surface term shifts the saturation density to 0.9$n_{0}$ and the energy per particle to $-13.9$ MeV. Values $\tilde{\epsilon}_{N}$ and $f_{0}$ are shown in Fig. 1 by short-dash lines. We see that $f_{0} < -1$ in some density interval. With this $f_{0}$ we solve Eq. (2) and obtain $f_{0}^{\text{tot}}$ and $\tilde{\epsilon}_{\text{tot}}^{(MF)} = \tilde{\epsilon}_{N} + E_{b}[f_{0}^{\text{tot}}]/n$ shown by solid lines. Finally long-dashed lines show the renormalized Landau parameter $f_{\text{ren,0}}$ and the corresponding energy per particle $\tilde{\epsilon}_{\text{tot}}(n) = E_{\text{tot}}(n)/n$ with $E_{\text{tot}}$ determined by