1. INTRODUCTION

The development of the technology of generating ultrashort electromagnetic pulses (USPs) raises an important problem of the characteristic features of their interaction with matter [1]. By now, single-cycle (one period at the carrier frequency) pulses with a duration of a few femtoseconds to tens of attoseconds have been experimentally generated in the visible and ultraviolet ranges [2], which provides new opportunities for superhigh time resolution spectroscopy. In the terahertz frequency range, half-cycle (half-period) pulses have been generated [3], which are promising, in particular, for quantum computations.

An important feature of the interaction of USPs with matter is that the probability of the photoprocess depends on the USP phase parameters such as the phase of the carrier frequency relative to the pulse envelope (the carrier-envelope, or CE, phase) and the coefficient characterizing the time variation in the frequency. The corresponding effects were investigated both experimentally and theoretically in [4–8]. Thus, Apolonski et al. [5] observed the dependence of the photocurrent from a gold foil irradiated by femtosecond laser pulses on the value of the CE phase, which can now be controlled with high accuracy [2]. In [6, 7], the effect of the CE phase on the excitation of matter by USPs was studied theoretically. In my earlier paper [8], phase effects occurring during the excitation of a Morse oscillator by an electromagnetic pulse the frequency of which varies linearly with time were analyzed numerically.

The objective of the present paper is to theoretically investigate the scattering of a USP in a plasma and to clarify how this process depends on the pulse duration, CE phase, and Debye radius.

2. BASIC FORMULAS

In perturbation theory, the effect of radiation on matter is usually described by using the cross section \( \sigma \), in terms of which the probability of the photoprocess in unit time is expressed as

\[
w = \sigma j,
\]

where \( j \) is the photon flux density. For sufficiently long pulses such that the radiation can be assumed to be quasi-monochromatic at the frequency \( \omega \), the photon flux density is \( j = I/\hbar \omega \), where \( I \) is the radiation intensity. For single- and sub-cycle electromagnetic pulses, the notions of the probability of a process in unit time and the radiation intensity are rather undefined. In this case, it is more adequate to use the notions of the probability \( W \) of the photoprocess throughout the pulse and the pulse shape, which is characterized by the dependence of the electric field strength on time, \( E(t) \), or on the frequency, \( E(\omega) \) (here, \( E(\omega) \) is the Fourier transform of \( E(t) \)).

In [9], the following expression was derived for the absorption probability throughout an entire USP:

\[
W = \frac{c}{(2\pi)^2} \int_0^\infty \sigma(\omega') |E(\omega')|^2 d\omega',
\]

where \( c \) is the speed of light and \( \sigma(\omega) \) is the photoabsorption cross section.

In [10], formula (2) was generalized to the case of scattering by an atom with allowance for the excitation of the atom and the nondipolar nature of the electromagnetic interaction. The approach developed in [10] can be used to describe the scattering of USPs in a plasma, provided that the atomic radiation scattering cross section is replaced with the corresponding plasma quantity.
Let us consider the scattering of a USP in a plasma. We assume that the spatiotemporal dependence of the electric field strength in the pulse has the form

$$E(t, r) = e E_0 g(t - \frac{n \cdot r}{c}),$$  \hspace{1cm} (3)$$

where $e$ is the unit polarization vector, $E_0$ is the electric field amplitude, $n$ is a unit vector in the pulse propagation direction, and the dimensionless function $g(\tau)$ is determined by the actual pulse parameters.

Using the formula that was derived in [10] for the probability of scattering of a USP by an atom in the high-frequency limit, we can obtain the following expression for the spectral-angular probability of radiation scattering in the plasma:

$$\frac{dW}{d\omega d\Omega'} = \frac{c E_0^2}{4\pi \hbar} \left[ 1 - (e \cdot n)^2 \right] r_c^2$$

$$\times \int \left[ g(\omega') \right]^2 S(\omega' - \omega, k' - k) \frac{d\omega'}{\omega}.$$ \hspace{1cm} (4)

Here, $r_c = e^2 / m_e c^2$ is the classical electron radius; $k = (\omega/c) n$, $n$ is a unit vector in the direction in which the radiation is scattered; $\Omega'$ is a solid angle in the direction of $n$; $\omega'$ and $k'$ are the frequency and wave vector of the scattered radiation;

$$S(\omega, k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{-i\omega t} \langle \hat{n}(k, t) \hat{n}(-k) \rangle.$$ \hspace{1cm} (5)

is the dynamic form-factor (DFF) of the plasma (the spectral density function [11]), which is replaced with the DFF of an atom from [10]; $\hat{n}(-k)$ is the spatial Fourier transform of the electron plasma density operator; and $\hat{n}(k, t)$ is the same spatial Fourier transform in the Heisenberg representation. The angle brackets in formula (5) denote both the quantum-mechanical and statistical averaging. The DFF of the electron plasma component can be represented as [12]

$$S(k) = \frac{|\tilde{\varepsilon}(k)|^2}{\varepsilon'} |\tilde{\varepsilon}_{ni}|^2 + Z_i \left| \frac{\varepsilon}{\varepsilon'} \right|^2 |\tilde{\varepsilon}_{ni}|^2,$$ \hspace{1cm} (6)

where $Z_i$ is the charge number of a plasma ion, $|\tilde{\varepsilon}_{ni}|^2 = \frac{n_e i}{\sqrt{2\pi v_T}} \exp \left( \frac{-\omega^2}{2k^2 v_T} \right)$

are the Fourier transforms of the squares of the thermal fluctuations of the electron and ion plasma components, $\varepsilon' = \varepsilon'(k)$ is the longitudinal component of the plasma dielectric function, and $v_T$ is the thermal velocity of the plasma particles. The superscripts ($i$) and ($e$) in the dielectric function and the same subscripts in the thermal velocity stand for the plasma electrons and ions. The electron dielectric function $\varepsilon^{(e)}$, which satisfies the inequality $\omega < \nu_T |k|$, describes the charge screening effect in the plasma. The corresponding expression has the form

$$\varepsilon^{(e)}(k) \equiv 1 + \frac{1}{d_e^2 |k|^2},$$ \hspace{1cm} (8)

where $d_e = \sqrt{T_e / 4\pi e^2 n_e}$ is the Debye radius of the plasma electrons with the density $n_e$ and temperature $T_e$. The ion contribution to the longitudinal plasma dielectric function can be ignored.

When substituted into expression (4) for the scattering probability, the first term in formula (6) describes the process in which the excess energy-momentum is transferred to a plasma electron. In the book by Ginzburg and Tsytovich [13], this scattering mechanism is called the Compton channel. The second term in formula (6) describes another channel for the process—the transition scattering, in which the excess energy-momentum is transferred to a plasma ion and the electromagnetic field is scattered by the plasma electrons inside the Debye sphere around the ion.

After averaging over the polarization of the incident radiation and integrating over the frequency of the scattered radiation, we can use formulas (4) and (6)–(8) to obtain the following angular distribution of the transition scattering of a USP per plasma ion:

$$\frac{dW_\epsilon}{N_i d\Omega'} = \frac{c E_0^2}{8\pi^2} Z_i \left( 1 + \cos^2 \theta \right)$$

$$\times \int \frac{|g(\omega')|^2 d\omega'}{\hbar \omega' \left[ 1 + (2d_e \omega' / \sin (\theta/2))^2 \right]^2}.$$ \hspace{1cm} (9)

A similar expression for the Compton scattering channel per plasma electron has the form

$$\frac{dW_\epsilon}{N_e d\Omega'} = \frac{c E_0^2}{8\pi^2} \left( 1 + \cos^2 \theta \right)$$

$$\times \int \frac{(2d_e (\omega' / \sin (\theta/2))^4 |g(\omega')|^2 d\omega'}{\hbar \omega' \left[ 1 + (2d_e \omega' / \sin (\theta/2))^2 \right]^2}.$$ \hspace{1cm} (10)

Expressions (9) and (10) have been derived by making the replacement

$$\exp \left( -\frac{\omega^2}{2k^2 v_T} \right) \rightarrow \delta(\omega),$$ \hspace{1cm} (11)

which is valid for $\omega \gg \nu_T |k|$, a condition that is assumed in the present paper. Replacement (11) implies that the scattered frequency $\omega'$ is equal to the Fourier expanded frequency $\omega$ of the incident USP ($\omega' \cong \omega$), and, accordingly, that inelastic processes occurring during the scattering can be ignored.