1. INTRODUCTION

In recent years, recycling and accumulation of hydrogen isotopes in the materials of the first wall of fusion reactors have received intensive study [1]. Interest in these problems is caused, first of all, by the fact that accumulation of a large amount of radioactive tritium in the construction materials of a fusion reactor makes the facility unsafe [2, 3]. The characteristic times of accumulation of hydrogen isotopes in the first-wall materials are usually much longer than the discharge duration. However, there are processes which proceed much faster. For example, uncontrolled release of a large amount of hydrogen is sometimes observed, which in some cases substantially affects the discharge and can even lead to its suppression [4]. The processes of hydrogen adsorption/desorption in the first-wall materials also affect the discharge dynamics during disruptions and ELMs. In particular, degassing of the wall during a disruption can result in a more than twofold increase in the plasma density [5]. Intense plasma energy fluxes onto the wall during ELMs are accompanied by a decrease in the plasma density, which is then restored to its previous value in the H-mode. The mechanism of these variations in the plasma density is still insufficiently understood, although it is clear that they are caused by the mutual effect of the recycling processes at the wall, including hydrogen absorption/desorption, and the processes occurring in the plasma [6]. Substantial thermal loads occurring during these processes can seriously damage the wall; moreover, an increase in the impurity concentration in the plasma due to erosion of the wall material can suppress the discharge completely.

In [7], it was shown in the framework of a zero-dimensional approximation that these phenomena can be explained by the development of thermal instability. This instability is caused by the positive feedback that appears when a slight increase in the wall temperature leads to an increase in the hydrogen desorption rate, which in turn leads to the growth of the plasma density and, accordingly, the radiative and charge-exchange energy losses, i.e., to an increase in the heat flux from the plasma onto the wall, thereby causing the further growth of the wall temperature. The conditions for the appearance of such feedback are determined by the ratio between the characteristic times of these processes. For example, if the characteristic time of hydrogen desorption from the wall is much longer (or shorter) than the energy and particles confinement times, then a fluctuation of one of the parameters of the system will result in oscillations that are damped as the system relaxes to its equilibrium state. Therefore, instability can develop only in a definite range of the plasma and wall parameters. In [7], a criterion for instability development was obtained using a model that assumed that the flux of hydrogen desorbed from the wall depended on the wall temperature according to the Boltzmann law; in turn, the wall temperature was assumed to be constant over the depth and was determined from the equation of total heat balance in the tokamak. Although the results obtained using this simplified model made it possible to estimate conditions for the development of thermal instability and its growth rate, the model was rather qualitative. Therefore, it is of interest to study thermal instability by using a more complicated model of plasma–wall interaction.

In this paper, we present a more detailed description of these processes. Variations in the plasma density and plasma energy are again determined from the zero-dimensional approximation, but the dynamics of the gas desorbed from the wall is described using a one-dimensional model that takes into account variations in the wall temperature with depth. The results of analytical solution are compared with numerical ones.
2. DISPERSION EQUATION

To describe hydrogen recycling in a tokamak, it is necessary to write out the equations describing variations in the plasma and wall parameters. As in [7], we will use the following balance equations for particles and energy in plasma (see also [8, 9]):

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + \frac{J A}{V},$$  \hspace{1cm} (1)

$$\frac{dW}{dt} = H - \frac{W}{\tau_E} - n^2 R,$$  \hspace{1cm} (2)

where $n$ is the plasma particle density; $W$ is the plasma energy density; $\tau_E$ and $\tau_p$ are the energy and particle confinement times, respectively; $H$ is the effective heating power per unit volume; $J$ is the particle flux from the wall into the plasma; $A$ and $V$ are the surface area and volume of the tokamak chamber, respectively; and $R$ is the coefficient characterizing charge-exchange and radiative plasma energy losses. In deriving these equations, it was assumed that the neutrals desorbed from the wall were instantly ionized and the gas evacuation (puffing) rate was much less than the rate of gas absorption/desorption by/from the wall.

Variations in the wall temperature $T(x,t)$ are described by the heat conduction equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2},$$  \hspace{1cm} (3)

where $\alpha$ is the thermal diffusivity, the coordinate $x = 0$ corresponds to the inner (plasma-facing) surface of the tokamak wall, and $x = L$ corresponds to its outer surface. The wall temperature on the inner surface is determined by the balance between the energy flux from the plasma, $V \left( H + \frac{W}{\tau_E} + n^2 R \right)$, and the heat flux in the wall material, $-\kappa \frac{\partial T}{\partial x} \bigg|_{x=0}$, where $\kappa$ is the thermal conductivity. We assume that the temperature at the outer surface is fixed and equal to $T_e$. Thus, the boundary conditions for Eq. (3) can be written as follows:

$$-\kappa \frac{\partial T}{\partial x} \bigg|_{x=0} = V \left( \frac{H}{\tau_E} + \frac{W}{\tau_E} + n^2 R \right), \quad T(x = L) = T_e.$$  \hspace{1cm} (4)

The equations describing the interaction of hydrogen with the wall should relate the densities of the particle fluxes from the wall, $J$, and onto the wall, $S = nV/A\tau_p$, to the wall temperature. It is well known that, after the wall material absorbs a certain amount of hydrogen, saturation occurs and the further irradiation of the wall by hydrogen does not lead to its additional accumulation. In saturated carbon, there are, on average, 0.4 hydrogen atoms per one carbon atom. For beryllium, this value is about 0.3 hydrogen atoms per one beryllium atom [10, 11]. In the saturation state, the distribution of hydrogen in the wall material within the penetration zone is almost uniform, the characteristic penetration depth being about 10 nm. Since we are interested in the stability of a steady-state solution, we can assume that the distribution of hydrogen over the depth is uniform. For small perturbations of the system parameters, the hydrogen content in the wall can be characterized by the accumulation $N$ (atoms/cm$^2$), which is independent of the depth. Due to high thermal diffusivity, which is on the order of 1 cm$^2$/s for the materials used in tokamaks (such as graphite, beryllium, and tungsten), temperature perturbations rapidly propagate over a distance much greater than the hydrogen penetration depth. Therefore, we can assume that the temperature within the hydrogen penetration zone is spatially uniform and equal to the temperature at the wall surface, $T_0 \equiv T(x = 0)$.

As the wall temperature grows, the rate of hydrogen desorption increases; however, the desorption mechanisms can differ substantially for different materials. For example, hydrogen absorption in graphite is mainly determined by the formation of chemical bounds between hydrogen and carbon atoms [10, 11, 12]. The increase in the temperature results in breaking of these bounds. Since diffusion of released atoms toward the surface and their recombination occur relatively rapidly, these processes insignificantly affect desorption. Therefore, variations in the amount of hydrogen accumulated in the wall can be described with good accuracy by the first-order desorption equation

$$\frac{dN}{dt} = -\frac{N}{\tau_i} \exp \left( \frac{E_i}{T_0} \right) + S \left( 1 - \frac{N}{N_m} \right) \equiv S - J,$$  \hspace{1cm} (5)

where $\tau_i$ is the characteristic desorption time and $E_i$ is binding energy of a hydrogen atom in the first-wall material. The second term on the right-hand side of Eq. (5) describes the increase in the amount of accumulated hydrogen due to the influx of particles from the plasma with allowance for saturation, and $N_m$ is the maximum amount of hydrogen that can be accumulated. As a rule, desorption is determined by several (rather than one) binding energies $E_i$, the typical values of which are in the range 1–3 eV. Therefore, Eq. (5) should contain the sum of several terms of the form $N \exp(-E_i/T_0)/\tau_i$, which correspond to the different energies $E_i$. To avoid sophisticated algebra, here, we restrict ourselves to only one such term, keeping this in mind when analyzing conditions for instability development.

Apparently, recycling of hydrogen in the beryllium wall can also be described by Eq. (5), because the processes of hydrogen interaction with graphite and beryllium have much in common [11].