Influence of Resonant Charge Exchange on the Viscosity of Partially Ionized Plasma in a Magnetic Field

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Abstract—The influence of resonant charge exchange for ion–atom interaction on the viscosity of partially ionized plasma embedded in the magnetic field is investigated. The general system of equations used to derive the viscosity coefficients for an arbitrary plasma component in the 21-moment approximation of Grad’s method is presented. The expressions for the coefficients of total and partial viscosities of a multicomponent partially ionized plasma in the magnetic field are obtained. As an example, the coefficients of the parallel and transverse viscosities for the ionic and neutral components of the partially ionized hydrogen plasma are calculated. It is shown that the account for resonant charge exchange can lead to a substantial change of the parallel and transverse viscosity of the plasma components in the region of low degrees of ionization on the order of 0.1.

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1. INTRODUCTION

Various software packages, e.g., B2-EIRENE, UEDGE-DEGAS, etc., are used to model edge plasma physics experiments. Numerical codes, describing the dynamics of charged particles, are based primarily on the well-known system of Braginskii equations for the fully ionized plasma [1], which incorporates the classic expressions for transport coefficients of ions and electrons, obtained by Braginskii. Meanwhile, for the modeling of the detached plasma in the divertor region of a tokamak, when the plasma degree of ionization is low and collisions of ions with atomic and molecular components dominate, the allowance for the influence of neutral particles on transport coefficients, in particular, on the coefficients of parallel and transverse plasma viscosity in the magnetic field, becomes important. The latter ones directly appear in the system of equations, describing the phenomena in the edge region of a tokamak [2, 3].

The theoretical expressions for the transport coefficients of the partially ionized plasma were obtained in the series of works on the basis of the solution of the Boltzmann’s kinetic equation by the Chapman–Cowling approximation [4]. By the example of hydrogen plasma, it was shown that the account for this effect leads to a sharp increase (almost on the order of magnitude) of viscosity of the ionic component at low degrees of plasma ionization. The general expressions for the viscosity of the partially ionized gas, corresponding to the second and third orders of the Chapman–Cowling approximation, were obtained by Devoto [9, 10]. The consideration of the second approximation for the fully ionized plasma leads thereby to the expressions, corresponding to the well-known results of Braginskii [1]. The allowance for the third approximation virtually has little effect on the specific numerical results.

General expressions for the components of the viscous stress tensor in the case of the fully ionized plasma in the magnetic field, corresponding to the second Chapman–Cowling approximation, were first obtained in [1]. For the partially ionized plasma in the magnetic field expressions for the respective quantities, derived in the 13-moment approximation of Grad’s method, were considered in [11, 12].

The general expressions for the viscous stress tensor of the partially ionized plasma in the magnetic field in the 21-moment approximation, which in terms of calculation accuracy correspond to the second Chapman–Cowling approximation and lead to the well-known results of Braginskii [1] for the fully ionized plasma, are obtained in the present work. In addition, the influence of resonant charge exchange is considered when calculating the parallel and transverse viscosity coefficients of the plasma components. Numerical dependences for the coefficients of partial and

1 The article was translated by the authors.
total viscosity of partially ionized hydrogen plasma on the degree of ionization at different plasma temperatures are found. The obtained results can be used in the analysis of transport processes in the edge region of a tokamak.

2. INITIAL EQUATIONS

The adequate description of transport processes in plasma embedded in the magnetic field is possible based on the solution of the kinetic equation using Grad’s method [6, 7]. We assume for simplicity that the temperature of the plasma heavy components is equal. Under these conditions the contribution of the electron component into the plasma viscosity can be neglected, thus only the heavy plasma components, ions and atoms, will be accounted for when calculating the viscosity coefficients below. In the 21-moment approximation the part of the general moment equations, which is written for the viscous stress tensor \( \Pi_{rs}^{(6)} \) and the auxiliary tensor \( \sigma_{rs}^{(0)} \), given by [7]

\[
-\omega_\beta \left\{ \pi_{rs}^{(0)} e_{sln}k_m \right\} + p_\beta W_{rs} = \sum_{\gamma} \left[ \frac{T}{m_\beta + m_\gamma} \left( G^{(0)}_{\beta\gamma} \pi_{rs}^{(0)} + G^{(2,4)}_{\beta\gamma} \pi_{rs}^{(2,4)} \right) \right] + \sum_{\gamma} \left[ \frac{\mu_{\beta\gamma}}{m_\beta + m_\gamma} \left( G^{(3,6)}_{\beta\gamma} \sigma_{rs}^{(3,6)} \right) \right],
\]

where \( \rho_\beta, p_\beta, \) and \( n_\beta \) are the mass density, pressure, and concentration of the particle species \( \beta \); \( T \) is the temperature expressed in energy units; \( \omega_\beta \) is the cyclotron frequency for the particle species \( \beta \); \( m_\beta \) is the mass of the particle species \( \beta \); \( \mu_{\beta\gamma} = \frac{m_\beta m_\gamma}{m_\beta + m_\gamma} \) is the reduced mass of particles of species \( \beta \) and \( \gamma \); \( k \) is the unity vector along the direction of the magnetic field; and \( e_{sln} \) is the Levi-Civita symbol. Herein, the parentheses on the left-hand sides of the equations correspond to the operation \( \{ K, L_r \} = \frac{1}{2} (K, L_r + K_r, L) - \frac{1}{3} \delta_{rs} K_r L_r \). The tensor \( W_{rs} \) is the shear velocity tensor and has the form \( W_{rs} = 2 \left( \frac{\partial u_r}{\partial x_s} \right) \), where \( \text{u} \) is the mass-averaged velocity of plasma.

The coefficients \( G^{(n)}_{\beta\gamma} \) for \( n = 1, 2 \), corresponding to the 13-moment approximation, were calculated for the multicomponent partially ionized plasma in [6]. The remaining coefficients that appear in the 21-moment approximation, were found in the analytic form only for the case of fully ionized plasma [6, 7]. In order to obtain all required coefficients \( G^{(n)}_{\beta\gamma} \) in the case of multicomponent partially ionized plasma, we consider the general system of equations for the tensorial moments of the distribution function, valid for any order of approximation of Grad’s method [7],

\[
\sum_{\gamma} \sum_{k=0}^{\xi-1} C^{2\text{nk}}_{\beta\gamma} n_\gamma \pi_{rs}^{(2)k} = -p_\beta W_{rs} + n_\beta \omega_\beta \left\{ b^{(0)20}_{\beta\gamma} e_{sln}k_1 \right\},
\]

\[
\sum_{\gamma} \sum_{k=0}^{\xi-1} C^{2\text{nk}}_{\beta\gamma} n_\gamma \pi_{rs}^{(2)k} = n_\beta \omega_\beta \left\{ b^{(0)2n}_{i\gamma} e_{sln}k_1 \right\},
\]

\[
1 \leq n \leq \xi - 1,
\]

where \( b^{(0)2n}_{i\gamma} \) are the moments of the distribution function and the coefficients \( A^{\text{mmk}}_{\beta\gamma} \) are expressed through the partial bracket integrals of Sonine polynomials \( S_{m}^{n} \),

\[
A^{2n}_{\beta\gamma} = n_\beta Q_{2nk} \varepsilon_{\frac{kn}{2}}^{-n} \times \left[ S_{\frac{nk}{2}}^{n} \left( W^2 \right) \left( W^2, W_s - \frac{1}{3} \delta_{rs} W^2 \right) \right]_{\beta\gamma}^{n},
\]

\[
B^{2n}_{\beta\gamma} = n_\beta Q_{2nk} \varepsilon_{\frac{kn}{2}}^{-n} \times \left[ S_{\frac{nk}{2}}^{n} \left( W^2 \right) \left( W^2, W_s - \frac{1}{3} \delta_{rs} W^2 \right) \right]_{\beta\gamma}^{n},
\]

where \( W = (m_\beta/2T)^{1/2} \), \( \varepsilon_{\beta} = -n \) is the dimensionless relative velocity of the particles of the species \( \beta \), \( \varepsilon_{\beta} \) is the velocity of the particles of the species \( \beta \), and \( \varepsilon_{\beta} = m_\beta/T \). The coefficients \( Q_{2nk} \) are defined as

\[
Q_{2nk} = (-1)^n \cdot 2^{nk} \cdot 4! n! (k + 2)! \cdot (2k + 5)!
\]

The general expressions for the partial bracket integrals and an example of calculation of one of them are given in the Appendix 1.