1. INTRODUCTION

The study of nanoobjects has been of constant interest for many years due to the prospects of using such materials in ultra-high-density memory devices and spintronics [1, 2]. Great expectations are associated with the applications of ferromagnetic nanoparticle suspensions in medicine, including the transport of pharmaceutical substances, the use of magnetocaloric effect for local impact on tissues, etc. [3].

Modern technologies can produce nanoparticles of various, in particular, triangular (see, e.g., [4, 5]) and square (rectangular) [6, 7] shapes, but the most popular are circular (elliptical) ones, which is associated with a vortex structure easily formed in such particles. In any case, the thickness of these objects is ultimately small, which allows considering them as two-dimensional (2D) materials and implementing the corresponding formalism for their analytical description.

Despite the prolonged investigation, the understanding of the processes occurring in nanodots has emerged only recently. This can be attributed to both the development of experimental techniques and an increase in the computational capabilities available to researchers. As became recently known, the magnetization reversal of nanodots is very difficult to describe analytically and numerical simulation is therefore an important tool in studying nonlinear dynamical processes [8–20].

Analytical estimates are based on the solution of the Landau—Lifshitz—Gilbert equation and its modification first described by Thiele [21]. The essence of the approach proposed in that work is as follows. The equation of motion of the magnetization in the presence of a magnetization inhomogeneity of the soliton type can be rewritten in terms of new collective variables, which are nothing more than the center-of-mass coordinate $X$ of the considered inhomogeneity.

In this case, the Thiele equation, as applied to the description of a magnetic vortex in a nanodot, takes the so-called “non-Newtonian” form [22]

$$\mathbf{G} \times \mathbf{v} + \nabla U + \hat{D} \mathbf{v} = 0.$$  (1)

Here, $\mathbf{G}$ is the gyrovector, $\mathbf{v}$ is the velocity vector of the magnetic vortex core, $U$ is the potential energy of the magnetization (its variation with the displacement of the vortex core from the center of the dot is associated with an increase in the magnetostatic energy), and $\hat{D}$ is the damping tensor. As is seen from Eq. (1), the vortex core is involved in complex motion with the presence of a gyroforce [23]. This indicates that its trajectory should be helical. This conclusion was confirmed many times by numerical simulations and attempts of the direct observation [6, 24, 25]. It should be noted that, in non-circular (non-elliptical) nanodots, the motion of the magnetization can have a more complex character, and the use of the Thiele equation is com-

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**On the Low-Frequency Resonance of Magnetic Vortices in Micro- and Nanodots**


Kirensky Institute of Physics, Siberian Branch of the Russian Academy of Sciences, Akademgorodok 50–38, Krasnoyarsk, 660036 Russia

Krasnoyarsk State Pedagogical University named after V. P. Astaf’ev, ul. Ady Lebedev 89, Krasnoyarsk, 660049 Russia

*e-mail: orlhome@rambler.ru*

Krasnoyarsk State Medical University named after Professor V. F. Voino–Yasenetski, ul. Partizana Zheleznyaka 1, Krasnoyarsk, 660022 Russia

Rzhanov Institute of Semiconductor Physics, Siberian Branch of the Russian Academy of Sciences, pr. Akademika Lavrentieva 13, Novosibirsk, 630090 Russia

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**Abstract**—The resonance motion of the magnetization of thin cylindrical and parallelepiped micro- and nanodots has been studied theoretically and experimentally. Analytical expressions for the external-field dependence of the resonance frequency of the vortex-structure oscillations have been derived taking into account the inertial and damping coefficients. The external-field dependence of the damping parameter has been found theoretically. The influence of the effective mass of a magnetic vortex on its low-frequency dynamics has been discussed.

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plicated [26]. As follows from Eq. (1), the characteristic frequency of the vortex rotation around the center of the nanodot neglecting dissipation is \( \Omega_0 = \kappa/G \), where \( \kappa \) is the effective stiffness of the magnetic sub-system.

Typical rotation frequencies of the vortex core depend on many factors (saturation magnetization of the material, its geometrical dimensions, shape, external field, etc.) and reach several hundreds of megahertz. Such a motion of the magnetization can be regarded as the low-frequency one. It is important to note that, in addition to the low-frequency magnetization modes in nanodots, Ivanov et al. [27–29] predicted the existence of more complicated high-frequency oscillations.

Investigation of the dynamical characteristics of the magnetization of nanodots is especially important in view of the implementation of these objects in ultrafast and power-saving memory devices. The quasi-static magnetization reversal of nanodots with a change in either the polarization vector or chirality is extremely difficult. In the former and latter cases, the work of the external field is mainly spent to surmount the exchange and magnetostatic energy, respectively. The natural solution is bringing the magnetic system to the resonance state with a subsequent flipping of the magnetization in the core. The method of such a “resonance revolving” of the magnetic vortex is quite successfully implemented in experiment and convincingly grounded in theory. The development of experimental techniques usually goes in two directions: (i) magnetization reversal by a short (nanosecond) pulse of the magnetic field [30–33] and (ii) magnetization reversal triggered by spin-polarized currents [34–37]. In any case, both analytical and numerical calculations are based on the solution of the Landau–Lifshitz–Gilbert or Thiele equations.

However, it should be noted hat this point that finding the general solution of Eq. (1) most closely approaching the reality is attended with great computational difficulties. Thus, researchers often resort to model representations.

There are tens of recent experimental works on the observation of the vortex motion. They revealed that the trajectory of the vortex core is more complicated than prescribed by Eq. (1). The presence of structural defects and vortex pinning failed to explain the distortions of the trajectory. In addition, resonance frequencies on the order of several gigahertz associated with the vortex motion but many times lower than predicted by Eq. (1) were discovered. Also predicted was splitting of the low-frequency mode into a doublet owing to the presence of inertial properties of the magnetic vortex.

The low-frequency regime of the core motion can be explained within the classical approach to the analysis of the Lagrangian of the magnetic system, in which one can separate the terms responsible for the kinetic energy of both rotational and translational motion of the core (see, e.g., [28, 29, 38]). Another direction of the analysis comes from adding phenomenological terms proportional to higher time derivatives of the vortex coordinate to Eq. (1): an inertial term [39] and a highest-order gyroscopic term proportional to the third derivative of the core coordinate [39–41].

The solution of Eq. (1) with the included inertial and highest-order gyroscopic terms differs from a smooth converging helix. In this case, the trajectory is a superposition of a smooth slow helical trajectory and high-frequency oscillations, whose shape resembles cycloids. Such a “fine” fast motion of the core on the background of a slow trajectory is thought to be responsible for the presence of high-frequency modes. To describe this motion, the Thiele equation should be written in the form

\[
G_j \times \dot{\mathbf{v}} + \mathbf{\mu}_\ast \mathbf{v} + G \mathbf{v} \times \nabla U + \dot{\mathbf{D}} = 0.
\]  

Here, \( \mathbf{\mu}_\ast \) is the effective mass tensor of the magnetic vortex.

The gyromagnetic vector can be written as \( \mathbf{G} = G \mathbf{z} \) where \( \mathbf{z} \) is the unit vector in the direction perpendicular to the nanodot plane. In the case of a 2D magnet, we can write

\[
G = \frac{M_s L}{\gamma} \left( \frac{\partial m_\perp}{\partial X_i} \frac{\partial m_\perp}{\partial X_j} - \frac{\partial m_\perp}{\partial X_i} \frac{\partial m_\perp}{\partial X_j} \right) dX_idX_j.
\]  

\[
D = -\frac{\alpha M_s L}{\gamma} \left( \frac{\partial m_\perp}{\partial X_i} \frac{\partial m_\perp}{\partial X_j} + \frac{\partial m_\perp}{\partial X_i} \frac{\partial m_\perp}{\partial X_j} \right) dX_idX_j.
\]

Here, \( \gamma \) is the gyromagnetic ratio, \( \alpha \) is the damping parameter, \( M_s \) is the saturation magnetization, \( L \) is the thickness of the magnet, which is much smaller than its radius \( R \). The smallness of \( L \) allows assuming that the magnetization does not change in the transverse direction.

The authors of the majority of theoretical works that we are aware of disregard the dissipative term owing to its smallness. This is indeed the case but the term responsible for damping can play a significant role at high velocities of the core motion (in the nonlinear regime) or in the analysis of the above high-frequency modes. In addition, this term affects experimentally studied resonance curves of nanodots and this fact focuses our interest on calculating the factor \( D \) and especially its dependence on the applied external field.

Rigorous calculation of the coefficients \( G_j \) and \( \mathbf{\mu}_\ast \) also remains a problem. As a rule, analytical calculations use approximate estimates. These two quantities depend on the profile of the magnetization distribution function \( \mathbf{M}(\mathbf{r}) \) in the vortex core, the exact expression for which cannot be derived.