INTRODUCTION

The electrical properties of continua at each point of space versus external field frequency $\omega$ are described by permittivity tensor $\varepsilon_{ij}$. However, in electrodynamics of continuous media, it is sometimes necessary to allow for, along with the frequency dispersion, the spatial dispersion, i.e., the dependence of tensor $\varepsilon_{ij}$ on wavevector $k$, $\varepsilon_{ij}(\omega) = \varepsilon_{ij}(\omega, k)$ [1]. The effects of spatial dispersion in the medium depend on ratio $\alpha/\lambda$, where $\alpha$ is some dimension characterizing the medium (e.g., lattice parameter, etc.) and $\lambda$ is the wavelength in the medium.

In optics, ratio $\alpha/\lambda$ is usually small and the spatial dispersion is observable only in macroscopic samples or when the electromagnetic wave propagates over a sufficiently long distance. However, allowance for the spatial dispersion is of great importance for understanding new effects, which are not observed in linear electrodynamics of continuous media. Among these effects are, for example, the optical activity, or gyrotropy (rotation of plane of polarization of an electromagnetic wave), and the emergence of “new” or “additional” waves, which have a wavevector other than that of the primary wave at the same frequency and polarization. Such new waves were first considered in works by Ginzburg [1] and Pekar [2].

In general, allowance for spatial dispersion makes related problems difficult to solve, since one must know the properties of tensor $\varepsilon_{ij}$ in this case. As far as we know, the effects of spatial dispersion in natural media have not been considered. First, they are weak, because ratio $\alpha/\lambda$ is small, and, second, the structural periodicity is atypical of natural objects because of the chaotic distribution of crystallites and other structural constituents in them. It might be expected that the effects of spatial dispersion will become still less observable at lower frequencies, because $\lambda$ grows.

However, it was reported [3] that new waves appear in the microwave (centimeter) range in the ice covering a freshwater lake. Significantly, they were observed in spring, when the ice temperature was close to 0°C. The new wave showed up through interference with the ordinary wave: when the distance between the receiver and transmitter was varied (in the experiment, from 40 to 100 m), the received power for the vertical and horizontal polarizations oscillated with the same spatial period rather than decreasing monotonically.

This work pursues an investigation into the effect of spatial dispersion in ice in order to find the reason for this effect and explain the earlier discovered anomalies in radio-sensed data for glacial bodies.

EFFECT OF SPATIAL DISPERSION

Qualitatively, the emergence of new waves can be understood by considering a solution to the problem, for example, for the electric component of a homogeneous monochromatic wave,

$$ E = E_0 \sin(\omega_0 t - \mathbf{k}_0 \cdot \mathbf{r}), $$

where $\omega = 2\pi/T$, $T$ is the oscillation period, $\mathbf{k} = 2\pi S/\lambda$, $S$ is the unit vector in the direction of wave propagation, and $\mathbf{r}$ is the radius vector.
If $E_0$ in Eq. (1) is modulated in time, \( E_0 = E' \sin \Omega t \), the electromagnetic wave contains two sinusoidal functions with frequencies $\omega_{1,2} = \omega_0 \pm \Omega$. If a medium with permittivity $\varepsilon_1$ contains a periodic structure with period $d$ and a permittivity other than $\varepsilon_1$, $E_0$ will be modulated in space, \( E_0 = E' \cos \Delta \kappa z \) (where $\Delta \kappa = 2\pi/d$ is a parameter characterizing a periodical structure), and two waves with spatial frequencies defined by the equations $\kappa_{1,2} = \kappa \pm \Delta \kappa$ will arise in space.

The existence of two waves with close $\kappa$ causes beats (see Fig. 1).

If a wave propagating in the medium is circularly polarized, beats will occur for orthogonally polarized waves and so four waves will coexist in the space [1]. In the general case, the values of $\kappa$ for the orthogonal fields will be different. The field structure for this case is shown in Fig. 2. At point $A$, the signal will be linearly polarized along the $x$ axis; at point $B$, along the $y$ axis. The spatial structure of the wave field illustrated in Fig. 2 allows us to predict some new effects.

1. **Variation of first Stokes parameter $S_1$ under external actions.** Assume that the positions of the receiver and transmitter remain unchanged but the properties of the medium vary, for example, because of the temperature dependence of $\varepsilon_j$. As a result, the structure shown in Fig. 2 will expand or shrink in the $z$ axis. The minimal value of $S_1$ will correspond to the fields near points $A$ and $B$. If some external parameter continuously increases and $S_1$ runs through several minima, the values of these fields can be used to estimate differences $\Delta \kappa$ between the wavenumbers of the waves observed. Feeble minima indicate that either shifts $\Delta \kappa$ between the new and ordinary waves are significant or the effect of spatial dispersion is weak.

2. **Anomalies in radar-sensed data.** In radar sensing of glaciers, the reflection from a thin ice crust may be intense or the echo signal from the interfaces between the media being sensed may disappear.

3. **Reversal of sign in the second, $S_2$, and third, $S_3$, Stokes parameters near points $A$ and $B$.** Remarkably, in the presence of spatial dispersion, this effect meshes with the attainment of minimal values by parameter $S_1$ (see Fig. 3).

4. **Variation in the angle of arrival of the signal with receiver–transmitter distance.** It was shown [1] that, in the presence of spatial dispersion, the group, $V_g$, and phase, $V_{ph}$, velocity vectors of the new wave do not coincide and make an angle greater than 90°. In general, this angle varies from 90° to 180°, depending on the type of anisotropy. If the distance between the transmitter and receiver is varied, the angle of arrival of the electromagnetic wave may change within a certain range according to the local anisotropy of the medium.

The negative dispersion of the new wave causes anomalies in the laws of refraction [4], which may generate a variety of exotic effects when the wave crosses interfaces.

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