Electrodynamic Properties of a Thin-Film Periodic Structure in an External Magnetic Field

A. A. Bulgakov\textsuperscript{a} and I. V. Fedorin\textsuperscript{b} *

\textsuperscript{a} Usikov Institute of Radiophysics and Electronics, National Academy of Sciences of Ukraine, ul. Akademika Proskury 12, Kharkov, 61085 Ukraine

\textsuperscript{b} National Technical University Kharkov Polytechnic Institute, ul. Frunze 21, Kharkov, 61002 Ukraine

* e-mail: iluxa617@yandex.ru

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Abstract—Reflection and transmission of light by a bounded periodically layered structure formed by periodically repeated dielectric and semiconductor layers in a magnetic field are studied. The period of the structure is assumed to be much shorter than the wavelength. The magnetic field vector is parallel to the layer plane, and the wave propagation direction is perpendicular to the magnetic field. Passage to the limit of the thin-layer structure is performed. It is shown that the structure in this case is a biaxial crystal. The properties of the reflection and transmission coefficients are studied.

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INTRODUCTION

Periodically layered media can be considered as a new type of artificial materials whose properties can be controlled because they depend on physical parameters of materials of which they are composed and on the geometrical size of the layers and the period of the structure \cite{1, 2}. Since spectral properties of periodic semiconductor materials depend on the external magnetic field, prospects of their practical applications become wider. In this case, the solution to the problem of wave propagation in this structure is determined by its configuration and by the wave propagation direction. Configuration of the structure is specified by two preferential directions: the periodicity direction and the orientation of the applied magnetic field.

This paper studies the medium consisting of alternating dielectric and semiconductor layers in an external magnetic field. The structure period is assumed to be much shorter than the wavelength. Unlike well-known works \cite{3, 4}, which consider a dielectric periodic structure, this paper addresses the thin-layer medium in an external magnetic field.

Ample literature is devoted to theoretical and experimental study of wave propagation in layered media \cite{2, 5, 6}. Investigation of electromagnetic wave reflection and transmission coefficients is one of the main experimental techniques for practical analysis of electromagnetic properties of optical structures. A procedure proposed in \cite{7} makes it possible to analyze the structure properties by the covariance method (with an arbitrary choice of the coordinate axes directions). This procedure is convenient to study anisotropic structures. Note that, at present, considerable interest is observed in such studies not only in the optical range, but also in the millimeter and submillimeter wave ranges.

It is shown in this paper that, optically, a thin-layer periodic structure is a biaxial crystal in which the permittivity parameters can be replaced with the effective permittivity, which depends on the layer thicknesses and on the magnetic field. Characteristic regions are found in the frequency dependence of the permittivity tensor components of the thin-layer structure. In these regions, singularities are observed in dependencies of the transmission and reflection coefficients on the frequency and magnetic field.

Analytic formulas are obtained for the transmission ($T$) and reflection ($R$) coefficients and calculations are performed for the electromagnetic wave incident on the bounded thin-layer periodic structure composed of dielectric and semiconductor layers in an external magnetic field. It is shown that the dependence on the incidence angle and on the external magnetic field can be used to find the pass and stop bands and to analyze the properties of the layers that compose the structure.

DISPERSION RELATION

Let us consider reflection and transmission of an electromagnetic wave in a bounded periodically layered structure formed by periodically repeated semiconductor and dielectric layers of thickness $d_1$ and $d_2$, respectively. It is supposed that the frequency of collisions in the semiconductor is $\nu = 0$ and losses in the dielectric can be neglected. The wavevector of the incident wave lies in the ($x$, 0, $z$) plane, and the $z$ axis is the periodicity axis. In this case, the problem can be significantly simplified in that dependence on the $y$ coordinate can be eliminated \cite{1}. The external mag-
netic field $H_0$ is applied in the direction of the $y$ axis (Fig. 1).

To solve the problem, we use the Maxwell equations in the semiconductor and dielectric layers and the continuity conditions for the tangential field components on the layer interfaces. For this geometry of the structure, the Maxwell equations split into expressions for two polarizations: the $E$ wave with $E_x$, $E_z$, $H_y$ field components (extraordinary waves) and the $H$ waves with $H_x$, $H_z$, $E_y$ field components (ordinary waves). Since expressions for the $H$ waves are similar to the homogeneous case and are independent of the external magnetic field, we further consider only the $E$ waves.

In the chosen coordinate system, the permittivity tensor for a semiconductor layer has the form [1]

$$
\varepsilon_{ij} = \begin{pmatrix}
\varepsilon_1 & 0 & i\varepsilon_3 \\
0 & \varepsilon_2 & 0 \\
-i\varepsilon_3 & 0 & \varepsilon_1
\end{pmatrix},
$$

(1)

where

$$
\varepsilon_1 = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2 - \omega_H^2} \right),
$$

$$
\varepsilon_2 = \varepsilon_0 \left(1 + \frac{\omega_p^2}{\omega^2 - \omega_H^2} \right),
$$

$$
\varepsilon_3 = \varepsilon_0 \frac{\omega_p \omega_H}{\omega (\omega^2 - \omega_H^2)},
$$

and

$$
\varepsilon_d = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2} \right).
$$

In these expressions, $\varepsilon_0$ is the permittivity of the semiconductor layer, $\omega_p$ is the plasma frequency of the semiconductor layer, and $\omega_H$ is the cyclotron frequency.

The dispersion relation for $E$ waves in the unbounded periodic structure, which relates $\omega$, $k_\|$, and $k$, has the form [1]

$$
\cos kd = \cos k_{z1}d_1 \cos k_{z2}d_2 - \frac{\varepsilon_d \varepsilon_{ij}}{2k_{z1}k_{z2}} \left(\frac{k_{z1}}{\varepsilon_j} e^{k_{z1}d_1} - k_{z2}e^{k_{z2}d_2} \right) \sin k_{z1}d_1 \sin k_{z2}d_2,
$$

(2)

where $k$ is the Bloch wave number, which characterizes periodicity of the structure;

$$
k_{z1} = \sqrt{\frac{\omega^2}{c^2} \varepsilon_j - k_x^2}, \quad k_{z2} = \sqrt{\frac{\omega^2}{c^2} \varepsilon_d - k_x^2}
$$

are the transverse wavenumbers of the semiconductor and dielectric layers;

$$
\varepsilon_j = \frac{\varepsilon_j^2 + \varepsilon_d^2}{\varepsilon_j}
$$

is the Voigt permittivity; $\varepsilon_d$ is the permittivity of the dielectric layer; and $d = d_1 + d_2$.

Let us consider the case when $k_{z1}d_1, k_{z2}d_2 \ll 1$. This actually means that the structure period is much shorter than the length of the electromagnetic wave in the direction of the $z$ axis. After expanding the trigonometric functions in Eq. (2) into series in small parameters, we introduce the following effective components of the permittivity tensor of the thin-layer structure:

$$
\varepsilon_{xx} = \frac{\omega}{\omega_0} b_1 (\omega a_1 - \omega b_2 - \omega_b),
$$

$$
\varepsilon_{zz} = \frac{\omega}{\omega_0} b_1 (\omega a_1 - \omega b_2 + \omega_b),
$$

(3)

where $a_i$ and $b_i$ are the coefficients given in the Appendix.

Now, the Bloch wavenumber plays the part of the transverse wavenumber of the structure as a whole:

$$
k = \sqrt{\frac{\omega^2}{c^2} \varepsilon_{xx} - \varepsilon_{zz}^2 k_{zz}^2}{\varepsilon_{xx}}.
$$

(4)

As follows from formulas (3) and (4), the thin-layer structure optically behaves as a biaxial crystal (the expression for the $\varepsilon_{yy}$ component is independent of the magnetic field) with the permittivity tensor components depending on physical parameters of the layers, layer thicknesses, and external magnetic field.

Let us consider the frequency dependence of the permittivity tensor components of the thin-layer structure. We will focus on the characteristic frequencies at which the tensor components vanish or tend to infinity. At the frequencies $\omega_{01,02}$, the components $\varepsilon_{xx}$ and $\varepsilon_{zz}$ simultaneously turn to zero ($\varepsilon_{xx} = \varepsilon_{zz} = 0$); at the frequencies $\omega_{\pm1,\pm2}$, the $\varepsilon_{zz}$ component turns to infinity; and at the frequency $\omega_g = \sqrt{\omega_H^2 + \omega_p^2}$ of the hybrid resonance, the $\varepsilon_{xx}$ component diverges.

1 Analytical expressions for $\omega_{01,02}^2$, $\omega_{\pm1,\pm2}^2$, $H_{01,02}^2$, and $H_{\pm1,\pm2}$ are provided in the Appendix.