On the Structure of Waves at the Charged Interface between Two Viscous Liquids at the Solid Bottom

A. V. Klimov, A. I. Grigor’ev*, and S. O. Shiryaeva

Demidov State University, Sovetskaya ul. 14, Yaroslavl, 150000 Russia

*e-mail: grig@uniyar.ac.ru

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Abstract—The solutions to the equation describing the wave motion in a bilayer system of immiscible liquids are obtained in the first order of the theory of approximations. The hydrodynamic potentials, current functions, generatrix of the shape, and electrostatic potential of the charged interface between two viscous liquids, one of which conducts current and the other being a dielectric, on the solid bottom, are determined. It is shown that in the case when the density of the upper medium is three or more orders of magnitude lower than that of the lower liquid or when the kinematic viscosity of the upper medium is negligibly low as compared to that of the lower medium, the effect of the upper medium on the flow of the liquid in the lower medium is negligibly small. The structure of the wave motion generated by the interface between the two media is analyzed.

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INTRODUCION

Analysis of physical regularities of the gravity—capillary—fluctuation wave motion in a plane finite-thickness layer of a viscous conducting liquid is important for many technical and technological applications [1—3]. A large number of earlier publications have been devoted to the capillary—gravity flow in layers of a viscous liquid [4—7], but the entire spectrum of the wave motion has not been covered. Nevertheless, it is well known [1—3] that in the short wavelength range (shorter that 10 nm [4]), it is the forces of fluctuation origin that are responsible for the generation of waves with the same dispersion relation as gravity waves [1, 3]. Fluctuation forces appear near the surface of solids (solid bottom or solid wall) at distances of ~100 nm and cause a change in the physicochemical properties of liquids [2, 8—10]. The structures of the flows (regularities of the distribution of the vortex and potential components of the velocity field over the layer thickness) associated with the periodic wave motion of the free surface of the finite-thickness layer of a viscous fluid was considered in [6]. It was found that the velocity field of the liquid flow associated with the capillary—gravity wave propagating over the free surface of a layer of a viscous liquid on a solid bottom has a complex structure. In this case, the vortex flow is concentrated in a small neighborhood of the free surface and in a small neighborhood of the bottom, while the potential flow fills the entire volume of the liquid. If the wavelength is much larger than the thickness of the liquid layer, the vortex flow generated by the surface wave fills the entire volume of the liquid, and the intensity of the vortex flow at the solid bottom may considerably exceed the flow intensity at the free surface of the liquid. If, however, the layer thickness is much larger than the wavelength, the vortex flow is concentrated at the free surface, and its intensity at the bottom tends to zero. This circumstance can be interpreted as the statement that the surface wave “does not feel the bottom.”

The structure of the flows associated with the periodic wave motion at the interface of two immiscible liquids was considered in [7]. It was found that the vortex flow is concentrated in small neighborhoods of the interface on both its sides in layers with a thickness on the order of tenths of the wavelength (depending on the viscosity of the liquid). It also turned out that the curls of the velocity fields suffer a discontinuity in the vicinity of the interface upon a transition through it. The structure of the gravity—capillary—fluctuation wave motion must obey the same regularities. It differs from the capillary—gravity wave motion only in that the capillary wave motion as such, with a typical dispersion relation \( \omega \propto k \alpha \propto k^{3/2} \), is limited by the motion with the dispersion relation \( \omega \propto \sqrt{k} \) (\( \omega \) is the wave frequency and \( k \) is its wavenumber) both on the side of long waves and on the side of short waves [1, 3, 11].

The features of the wave motion in thin layers of liquid (with thickness \( h \leq 100 \text{ nm} \)), in which the effect of fluctuation forces becomes significant [8—10], were considered in [1, 3, 11]. An important result of calculations in the linear approximation is that the component in the dispersion relation generated by the action of gravity and fluctuation forces have the same dependence on the wavenumber and appear in this relation as a sum. In nonlinear computations, it was found that the wave motion of the liquid appearing as a result of the nonlinear interaction as a correction to the waves...
defined at the initial instant are expelled by the field of fluctuation forces to the periphery of the domain of influence of these forces. The value of the nonlinear corrections to the wave amplitude depends on the electric field at the free surface of the liquid because the nonlinear effects lead to an increase in the curvature of field vertices in the presence of an electric field and to a decrease in their curvature in zero field.

In this study, we pay attention to the circumstance that the solid bottom that produces the field of fluctuation forces in the liquid modifies its properties in a thin layer of ~100 nm adjoining the bottom, which lies on the boundary of the domain of action of short-range fluctuation forces [8–10]. The deformation of this boundary leads to the emergence of internal wave motion in the liquid. The features of such wave motion will be analyzed in this study.

It should be borne in mind that the gravity—capillary—fluctuation waves do not exist in actual practice, and we are dealing either with gravity—capillary or capillary—fluctuation waves. However, it is convenient to analyze these waves together as individual branches of the continuous spectrum of the gravity—capillary—fluctuation motion.

1. FORMULATION OF THE PROBLEM

Let us consider two viscous incompressible liquids. One of the liquids with density \( \rho_1 \) and viscosity \( \nu_1 \) forms an infinitely large layer in the field of gravity forces, which is described by the geometrical locus \(-d \leq z \leq 0\), where \( d \) is the layer thickness. The layer of the first liquid rests on a solid bottom. The second liquid with density \( \rho_2 \) and viscosity \( \nu_2 \) fills the half-space \( z > 0 \) above the layer of the first liquid. Analysis is carried out in the Cartesian system of coordinates with the \( z \) axis directed against the acceleration due to gravity, \( \mathbf{e}_z \parallel -\mathbf{g} \), and the \( x \) axis coincides with the direction of propagation of a plane wave \( \exp(st - ikx) \) (here, \( \mathbf{e}_x \) is the unit vector of the \( x \) axis, \( s \) is the complex frequency of the wave, \( k \) is the wavenumber, \( t \) is the time, and \( i \) is the imaginary unity).

We assume that the upper liquid is a dielectric with permittivity \( \varepsilon \) and the lower liquid is a perfect conductor. The interface between the media is characterized by surface tension \( \gamma \). We assume that plane \( z = 0 \) coincides with the unperturbed interface between the media, over which an electric charge is distributed uniformly so that a uniform electrostatic field \( E_0 \parallel \mathbf{e}_z \) exists in the upper liquid in the absence of deformation of the interface.

In accordance with the prevailing theoretical concepts, molecules of the liquid close to the solid wall have limited mobility and experience the ordering action of the wall, and dipole molecules experience the orienting action also [8, 12]. For example, experiments [2, 8, 12] show that in thin water layers with a thickness of ~100 nm, the effect of the solid wall leads to an increase in the viscosity of the liquid (by approximately 1.5–2 times), its density (by approximately 2%), and reduces the permittivity (to approximately 1/3). The reason for such a change in the physical properties of water is most probably the change in its structure (change in the system of hydrogen bonds), which becomes more ordered near the solid wall. Numerical model experiments lead to the same results.

Let us now consider the modified thin layer of liquid at the solid wall, in which molecules are ordered and have a preferred orientation, as a result of the interaction with the wall. The density and viscosity of the liquid in the near-wall layer exceed the corresponding values in the bulk of the liquid at a large distance from the wall [2, 8, 12]. The same mechanisms that are responsible for the formation of a double electric diffusive layer at the free surface of the liquid operate in the near-wall layer also. Thus, we can assume that the near-wall layer of the liquid is separated from its bulk by a certain interface with a nonzero interfacial tension [8, 12]. A peculiar feature of the liquid in the near-wall region is that it is in the field of fluctuation forces generated by the wall. Therefore, the wave motion in the near-wall layer of the liquid, which is excited by the interface, appears when fluctuation forces play the leading role.

This means that fluctuation forces strongly depending on the distance are exerted by the solid bottom on each liquid particle in the layer. For qualitative estimates, we assume that these forces are inversely proportional to the third power of distance \( l \) between a given liquid particle and the solid bottom; for a liquid layer of thickness \( d \), we can write this relation in the form \( [1–3, 8–10, 12] F \propto l^{-3} \).

The pressure exerted by the fluctuation forces on the interface \( z = \xi(x, t) \) between the media perturbed by the capillary wave motion is defined as \( F = A(6\pi(d + \xi)^3)^{-1} \). Actually, exponent “3” in this expression depends on the distance and varies from three to four upon an increase in this distance [8, 12]. For subsequent qualitative estimates, we assume that proportionality factor \( A \) with the dimension of energy for a layer of a liquid on the surface of a solid is \( 10^{-13} \text{ erg} \). (According to [8], the value of constant \( A \) is \( 1.2 \times 10^{-13} \text{ erg} \) for water in contact with quartz and \( 1.53 \times 10^{-13} \text{ erg} \) for water in contact with mica.)

Let us suppose that at initial instant \( t = 0 \), the interface between the liquids is deformed by a plane capillary—gravity wave \( \xi(x, t) = \xi_0 \exp(st - ikx) \), whose amplitude \( \xi_0 \) is modulo much smaller than the capillary constant of the lower liquid: \( |\xi_0| \ll \alpha_1 = \sqrt{\gamma/(g \rho_1)} \) (in the general case, amplitude \( \xi_0 \) is assumed to be complex-valued). The velocity fields of the flows of the liquid in the upper and lower media, which are associated with the wave, will be denoted by \( U^{(1)}(r, t) \) and \( U^{(2)}(r, t) \), respectively.

We assume that velocity fields \( U^{(1)}(r, t) \) and \( U^{(2)}(r, t) \) in dimensionless variables (e.g., in such that \( \rho_1 = \gamma = g +... \)