INTRODUCTION

Electromagnetic nonlinear dispersive transmission lines are of considerable interest both from the theoretical standpoint and for applications. Monograph [1] mainly describes the first stage in the evolution of the theory of nonlinear waves and its applications in radio physics and electronics. The review of experimental works of the first stage can be found in [2].

The lumped LC transmission lines with specially selected dispersion and nonlinear characteristics in the megahertz rf range are ideal models of nonlinear dispersive media. Such transmission lines ensure the variety of dispersion and nonlinear characteristics of the media, which can be obtained by simple selection of elements themselves, their parameters, and even their mutual arrangement. For example, it was shown in [3] that the proposed lumped transmission line is an electric model of a 1D mechanical anharmonic chain of atoms in which the interaction of particles is determined by the Morse potential (Toda lattice). It is well known in biophysics that biological membranes and nerve fibers can be described as nonlinear two-wire lines with an active element resembling the p—n junction [4].

Various radio-engineering applications of electromagnetic solitons are described in [5]. Investigations of the direct transformation of a video pulse into a radio pulse during its propagation in electromagnetic lines are being actively carried out. Generation of a train of solitons in LC chains with a nonlinear capacitance was demonstrated in [6]. A soliton generator based on an electromagnetic transmission line was described in [7]. The possibility of direct and effective transformation of a video pulse into a radio pulse during its propagation in ferrite-based lines with spatial dispersion and nonlinearity was investigated in [8].

A new method for transforming a high-voltage video pulse (110–290 kV) into rf oscillations (0.6–1.1 GHz) in a nonlinear transmission line with temporal dispersion ensured by pulsed reversal magnetization of ferrite was studied experimentally [9] and theoretically [10].

In one of the first publications devoted to analysis of solitary waves in electric circuits [11], the properties of Korteweg–de Vries (KdV) solitons were studied experimentally and theoretically mainly for a quadratic nonlinearity. However, solitary waves for a cubic nonlinearity of the line have not been considered in [11] and in any of the subsequent publications. In this study, a cubic nonlinearity is realized on two p—n diodes connected reversely in parallel. In addition, solitons based on more exact model equations, viz., the nonlinear transmission line equation (NTLE), double dispersion equation (DDE), and Boussinesq equations, have not been studied. Ohmic loss in electromagnetic lines have not been studied either. This work is aimed at filling this gap.

Video pulses that preserve their shape during propagation in a nonlinear dispersed transmission line are referred to as video solitons like in [12].

1. PHYSICAL FORMULATION

A discrete transmission line of the type of a low-frequency filter with semiconductor diodes (the equivalent circuit is shown in Fig. 1) was chosen as a nonlinear dispersive wave system. Each section of the line is an LC four-pole in which a reverse-biased p—n diode was chosen as a nonlinear capacitance. Here, \( n \), \( n + 1 \), \( n + 2 \), ... are the numbers of sections (nodes) of the line, \( L \) is the linear inductance, \( R_1 \) is the ohmic resistance of the rectifying layer of the p—n junction, \( R_2 \) is
the resistance of the crystal of the base region plus the resistance of the contact or a series resistance for studying dissipation effects, and \( C_1 \) is the correcting capacitance that can be connected to a section \((L)\) or in it. To simplify the diagram, we do not show the constant bias circuit intended for selecting the working point.

The nonlinearity of the \( LC \) circuit is due to capacitance \( C(u) \), which is a function of the bias voltage and signal voltages in the parallel branch. The dispersion of the transmission line is associated with the discreteness of the parameters of the line (spatial dispersion) and the presence of constant correcting capacitance \( C_1 \) (temporal dispersion). Apart from nonlinearity, solitons appear due to a certain type of linear dispersion, for which the medium should be nondispersive in the vicinity of zero frequency for the given type of normal waves. It is this type of dispersion (see relation (7) below) that is observed in the transmission line under investigation.

1.1. Capacitance of the \( p–n \) Junction

As a nonlinear capacitance, we can use semiconductor diodes (stabilitrons and varicaps), the gate-source \( p–n \) junction of a field transistor, a heterojunction, etc. The nonlinear capacitance of the \( p–n \) junction is the sum of two quantities: the capacitance of the rectifying layer (barrier capacitance) and the diffusion capacitance of minority carriers. The barrier capacitance is due to static charges of ionized impurities. The double layer of unlike electric charges is similar to a parallel-plate capacitor, and its capacitance is defined by the formula [13]

\[
C_s(U) = C_{0s} \left(1 - \frac{u}{\phi_k}\right)^{-\gamma},
\]

where \( u \) is the external voltage applied to the \( p–n \) junction and \( \phi_k \) is the potential associated with the barrier at the junction (usually amounting to 0.4–0.9 V). Constant \( C_{0s} \) is the capacitance corresponding to zero voltage (which is from several tens up to several hundred pF) and \( \gamma \) is the nonlinearity index of the junction, which is determined by the distribution of impurities in the junction. For diodes with an abrupt junction (uniform impurity distribution), we have \( \gamma = 0.5 \). For diodes with a diffusion junction (linear impurity distribution), \( \gamma = 1/3 \). Formula (1) holds for reverse voltages lower than the breakdown voltage.

For a direct voltage across the diode, diffusion capacitance \( C_s \) associated with injection, diffusion, and recombination of minority carriers appears in addition to barrier capacitance \( C_s \). When direct current is passed through the \( p–n \) junction, electrons are injected to the \( p \) region and holes are injected to the \( n \) region. Simultaneously with the injection, the number of majority carriers increases in the \( p \) and \( n \) regions by virtue of the electroneutrality condition. The charges of holes injected to the \( n \) region are neutralized by the charges of additional electrons supplied to the \( n \) region from the external circuit so that the space charge density in the \( n \) region remains at the zero level. Analogous processes occur in the \( p \) region.

Capacitance \( C_s \) strongly depends on the voltage across the \( p–n \) junction (follows the exponential law). If the characteristic time of the process is shorter than the lifetime of charge carriers, we have

\[
C_s(u) = C_{0s} \exp \left( \frac{u}{\phi} \right),
\]

where \( \phi = kT/e \) is the temperature potential. The total capacitance of the \( p–n \) junction is the sum of these capacitances \( C(u) = C_s + C_j \). After locking of the junction, \( C_s \) rapidly decreases to zero; the injection of the minority carriers ceases at reverse voltage \( u > (4–6)\phi \), and \( C_j \) almost vanishes. Capacitance \( C_s \) is shunted by the resistance of the unlocked junction, and its Q factor is therefore small (usually smaller than unity in the entire frequency range).

Figure 2 shows the typical capacitance–voltage \((C–V)\) characteristic of the varicap diode with the following parameters: \( C_{0s} = 6 \times 10^{-13} \text{ F} \), \( \phi_k = 0.6 \text{ V} \), \( C_{0s} = 6 \times 10^{-10} \text{ F} \), and \( \phi = 0.04 \text{ V} \). In most cases, the dependence of charge \( Q(u) \) on the nonlinear capacitor can be approximated by the cubic polynomial

\[
Q(u) = C_0 u + C_{N_1} u^2 + C_{N_2} u^3,
\]

where coefficients \( C_0 \), \( C_{N_1} \), and \( C_{N_2} \) depend on the type of the diode. For selecting the working point, a constant blocking voltage \( V_b \) is applied.

Let us consider a dispersive transmission line with the third-order nonlinearity:

\[
Q(u) = C_b u \pm C_{N_2} u^3.
\]

The negative cubic nonlinearity \("–"\) is observed for low voltages across the capacitor with a ferroelectric at a temperature above the Curie point.