INTRODUCTION

The wave equation is an almost exact macroscopic mathematical model of the electromagnetic field, which can be used for solving a number of topical problems in many technical applications. It is well known [1, 2] that the rigorous analytic solutions to the wave equation at frequency \( \omega > 0 \) for the electromagnetic field in a layered medium is described by the integral on the real half-axis of the complex-valued function, which is known as the Sommerfeld solution.

The methods for evaluating this integral are well known. The integral is evaluated approximately, for which various numerical algorithms or asymptotic methods (in most cases, the steepest descent method), or their combinations are used. For solving many applied problems (and especially in scientific investigations), such methods are quite admissible. However, omitting detailed analysis of the known solutions and generalizing the available results, we can state that the Sommerfeld solution can hardly be used for some technical applications (in particular, for solving design problems in radio electronics, e.g., for estimating crosstalk disturbance quantitatively for which the field has sometimes to be calculated hundreds and thousands of times). When commonly used computers are employed, the Sommerfeld solution is certainly inapplicable in view of the extremely long computer time required for this purpose. The well-known methods for calculating fields based on spatial discretization of the object being simulated on a fine mesh (finite-element method, method of moments, etc. [3]) suffer from the same drawback. The situation cannot be changed appreciably either by using the method of partial element equivalent circuit of equivalent circuit (PEEC method) [4, 5], which can be assumed to be developed completely. A pragmatic (“technical”) approach to solving this problem is required, which would give a solution in theoretical physics that is not always suitable for applications.

1. PROPOSED METHOD

The computational capacity of the problems involving the calculation of the field can be reduced considerably by using the electrodynamic approach based on the equivalent propagation constant (EPC) method proposed in [6]. The basic idea of the method is that Green’s function \( G \), which is the solution to the wave equation, can be described for a layered medium by an expression of the same type as for a homogeneous medium:

\[
G = M \exp(-ik_{\text{epc}} R) / R,
\]

where \( M \) is the amplitude factor, \( k_{\text{epc}} = \omega \sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_e} \) is the EPC, \( R \) is the distance between the point at which the field is calculated and the point of location of an elementary source of the field, \( \varepsilon_0 = 10^{-9}/(36\pi) \) is the Coulomb constant, \( \mu_0 = 4\pi 10^{-7} \) is the Biot–Savart constant, \( \varepsilon_r \) and \( \mu_r \) are the equivalent permittivity and permeability of the layered medium, respectively, \( \omega \) is the angular frequency, and \( i \) is the imaginary unit.

However, it is expedient to simplify the expressions for \( \varepsilon_r \) and \( \mu_r \) proposed in [6]. The modified formulas have the form

\[
\varepsilon_r(r) = \frac{1}{R} \int_0^\infty F_0(\lambda, r) F_0(\lambda) d\lambda,
\]

\[
\mu_r(r) = R \int_0^\infty F_0(\lambda, r) \Phi_0(\lambda) d\lambda,
\]

where \( F_0(\lambda) \) is the mathematical model of a layered medium obtained in the solution of the electrostatic problem, which corresponds to the structure of the
Bessel function of the first kind; \( \Phi_0(\lambda) \) is the mathematical model of the same medium obtained from the solution of the magnetostatic model; \( J_0 \) is the zeroth-order Bessel function of the first kind; \( r = \sqrt{(x-x_0)^2+(y-y_0)^2} \); \( x \), \( y \), and \( z \) are the abscissa, ordinate, and applicate of the point at which the field is calculated, and \( x_0 \), \( y_0 \), and \( z_0 \) are the abscissa, ordinate, and applicate of the point of location of the field source; improper integrals are evaluated using the technique described in [8].

With such a definition of \( \epsilon_r \) and \( \mu_r \), these quantities are independent of the size of the source and of its shape, which considerably reduces the required computer time as compared to the version proposed in [6].

The idea of describing an electromagnetic process by a mathematical model in which one of the parameters is calculated in the quasi-stationary approximation is not new. This approach was used, for example, for describing electromagnetic processes in transmission lines with distributed parameters by the Helmholtz equation [9]. The solution to this equation is well known. It describes the propagation of the incident and reflected waves in the channel of propagation of the electromagnetic energy (which includes a conductor) and is the sum of two terms with an exponential dependence on the distance. The exponents differ in sign and can be calculated in terms of the distributed parameters of the line, including the capacitance and inductance that can be calculated by solving the Laplace equations for the electric and magnetic field potentials (i.e., strictly speaking, in direct current). Nevertheless, the resultant mathematical model is effectively used in a wide frequency range. On this basis, a powerful theory was constructed, which is known as the theory of a line with distributed parameters and has a wide and well-known range of correct application. The correctness of using this approach for describing analogous processes in the channel of propagation of the electromagnetic energy (containing no conductor) was discussed in [10]. Let us consider this problem in greater detail by estimating the correctness of mathematical model (1) for the problems permitting a rigorous solution.

2. VERTICAL DIPOLE IN A LAYERED MEDIUM

It is well known [2] that the rigorous solution of the wave equation for vector potential \( A_v \), of the field produced by a vertical current element (vertical elementary dipole) in the \( v \)th layer of the medium with plane-parallel interfaces between the layers at a frequency \( \omega > 0 \) is described by the integral

\[
A_v(r, z, z_0, \omega) = M \int_0^\infty J_0(\lambda r) \Phi_v(\lambda, z, z_0, \omega) d\lambda, 
\]

where complex function \( |\Phi_v(\lambda, z, z_0, \omega)\rangle \) is the mathematical model of the layered medium [7], which is determined at frequency \( \omega \) from the boundary conditions for the vector potential at the interfaces between the layers, and \( M \) is the amplitude factor.

In the quasi-static case for \( \omega \rightarrow 0 \), we have

\[
A_{0v}(r, z, z_0) = M \int_0^\infty J_0(\lambda r) \Phi_{0v}(\lambda, z, z_0) d\lambda, 
\]

where \( \Phi_{0v}(\lambda, z, z_0) = \lim_{\omega \rightarrow 0} \Phi_v(\lambda, z, z_0, \omega) \).

Using the simplest identity transformation, we can write this expression in the form

\[
A_{0v}(r, z, z_0) = \frac{M}{R} \int_0^\infty \left| J_0(\lambda r) \Phi_{0v}(\lambda, z, z_0) \right| d\lambda, 
\]

Since by definition (2) the expression in the square brackets for a dipole is the equivalent permeability \( \mu_e \) adopted in the EPC method, we have

\[
A_{0v}(r, z, z_0) = \frac{M \mu_e}{\mu_0}. 
\]

Thus, for identical notations, the expression used for describing the field in the EPC method coincides with the known expression for the vector potential in direct current, which is the solution to the Laplace equation. An analogous conclusion can be drawn for a scalar electric field potential, the only difference being in the notation. Consequently, we can assume that the EPC in direct current gives exact values for the field. This means that the method gives the most accurate results in the range of lower (but not low) frequencies for which its correctness is indisputable. Computational experiments have shown that the working frequencies of the method can amount to several units, tens, and even hundreds of gigahertz.

3. VERTICAL DIPOLE ON A PERFECTLY CONDUCTING PLANE

It is well known that the solution to the wave equation for the vector potential of a vertical elementary dipole on a perfectly conducting plane with free space above it has the form [2]

\[
A_v = 2 R \frac{M e^{-i k_0 R}}{k_0}, 
\]

where \( k_0 = \omega \sqrt{\varepsilon_0 \mu_0} \) is the constant of propagation of an electromagnetic wave in the upper (free) half-space.

For direct current also (frequency \( \omega = 0 \)), we have

\[
A_v = \frac{2 M}{R}. 
\]

Therefore, the equivalent relative permeability which by definition is the ratio of the magnetic field