Electric-Field Control of Electron Interference Effects in 2D Semiconductor Nanostructures with Parabolic Quantum Wells

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Abstract—The possibility of the influence of a constant electric field on spatial reproduction effects for quantum-mechanical current density, which arise during interference of electron waves in 2D semiconductor nanostructures, is studied theoretically. It is found that, in structures comprising a narrow rectangular quantum well and a wide parabolic quantum well placed successively in the direction of propagation of an electron wave, the transverse current density distribution existing at the input of the wide quantum well is reproduced in this well with a certain accuracy in periodically repeated cross sections. The inhomogeneous current density distribution arises because of the interference of electron waves propagating in the wide quantum well simultaneously in different quantum-dimensional subbands. It is shown that it is possible to control these effects via the use of a constant electric field perpendicular to the axis of the structure in the region of the wide quantum well.

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INTRODUCTION

At present, the advances in nanotechnology have allowed creation of semiconductor nanostructures in which the linear dimensions of a 1D or 2D conducting channel in the direction of propagation of an electron wave are smaller than an electron’s free path. In this channel, particles move in the ballistic regime, which allows experimental study of ballistic transport effects in these structures, in particular, various electron interference effects. Theoretical basics of these effects and the analysis of basic experimental results in this field are given in a number of monographs [1–3]. There are many theoretical studies devoted to the investigation of ballistic electron transport in 1D and 2D nanostructures whose common specific feature is the presence of regions with sharp (nonadiabatic) variation in either the channel geometry or the potential profile in quantum channels. For example, quantum transport was theoretically studied in 1D channels of rectangular [5] and parabolic [6] shape that connect two 2D electron reservoirs, in different types of quantum point contacts connecting these reservoirs [7], in \( \perp \)-shaped channels [8, 9], in channels with sharp bends and curved channels [10–12], in channels with a \( \delta \)-like scattering center inside [13], in crossed channels [14], in single geometrically inhomogeneous channels with sections of different width [15–18], and in geometrically homogeneous 1D and 2D nanostructures with sections of sharp variation of the potential profile controlled by a constant transverse electric field [19]. The role of damped modes in quantum point contacts was considered in [13, 20, 21]. However, in all these studies (and a number of other studies in this field), the final objective is the calculation of either the transmission coefficient or the conductance of the quantum-mechanical structure. Since finding these quantities requires calculation of the total current of particles in the quantum channel, a goal that is achieved via integration of the coordinate-dependent quantum-mechanical current density over the cross section of the channel, it is natural that, in such studies, spatially inhomogeneous effects related to the quantum-mechanical current density are absent.

The main objective of this study is the theoretical investigation into the possibility of applying a constant transverse electric field with strength \( F \) for control of the effects of spatial inhomogeneity for the probability current density \( j_{x}(x,z) \) (or quantum-mechanical current density \( ej_{x} \) ) arising in 2D semiconductor nanostructures shaped as a narrow quantum well (QW\(_{1}\)) rectangular along the \( z \) axis (the \( z \) axis is the axis of dimensional quantization) and a wide quantum well (QW\(_{2}\)). Both wells are situated successively along the direction of propagation of an electron wave (the \( x \) axis). Let us show that, owing to interference of electron waves propagating in wide quantum well QW\(_{2}\) simultaneously in several quantum-dimensional subbands, inhomogeneous distribution \( j_{x}(x,z) \) appears. In this case, transverse distribution \( j_{x}(0,z) \) at the input of wide quantum well QW\(_{2}\) is reproduced with a certain accuracy at distance \( X_{1} \) from the input (reproduction). In this case, initial distribution \( j_{x}(0,z) \) is periodically reproduced in the
cross sections $X_p = pX_1$ (where $p$ are integer numbers). We recall that, for a free particle, $j_z$ is independent of coordinates [4].

Earlier, possible reproduction effects for electron waves [22] and the existence of the effects of reproduction and multiplication for $j_z(x, z)$ in a symmetric 2D semiconductor nanostructure were briefly discussed [23]. In [24, 25], we presented detailed analysis of these effects in 2D semiconductor nanostructures based on parabolic [24] and rectangular [25] quantum wells with particular geometric parameters for a GaAs–GaAlAs system.

1. MODEL AND CALCULATION TECHNIQUE

Let us present the model and the calculation technique. We consider a symmetric (Fig. 1a) (or asymmetric, Fig. 1b) 2D nanostructure consisting of two quantum wells successively situated along the $x$ axis: quantum well $QW_1$ with potential $U_1(z)$ (at $x < 0$) and quantum well $QW_2$ with potential $U_2(z)$ (at $x > 0$). The wells localize a particle along the $z$ axis (or along the normal to the well planes). It is assumed that the motion along the $y$ axis is free and that the potential energy within each of the regions is independent of $x$ and changes stepwise at the point where the wells are joined ($x = 0$). Effective masses $m^*$ of the particles are assumed to be isotropic and equal in both regions. Then the Schrödinger equations describing the motion of the particles along the $z$ axis in each of the regions have the form

$$-\frac{\hbar^2}{2m^*} \frac{d^2\psi_n(z)}{dz^2} + U(z)\psi_n(z) = E_n\psi_n(z).$$

Here, $E_j$ and $E_n$ are the eigenvalues and $\psi_n(z)$ are the eigenfunctions of Eqs. (1) and (2), respectively, in $QW_1$ and $QW_2$. The total energy of the particle is $E = E_{x,z} + E_y$, where $E_y = \hbar^2 k_y^2/2m^*$ is the energy corresponding to the free motion along the $y$ axis. Let us consider the situation in which a monochromatic electron wave of unit amplitude propagates from left to right from $QW_1$ to $QW_2$ in quantum subband $m$. It will be assumed that the quantum wells localizing the particle along the $z$ axis have infinitely high potential barriers; i.e., in this direction, the energy spectra are completely discrete in both wells. Then, wave functions $\Phi^{(1)}(x, z)$ and $\Phi^{(2)}(x, z)$ of the particle in each of the regions separately have the form

$$\Phi^{(1)}(x, z) = \psi_m(z)\exp(ik_{nx}x) + \sum_j B_j\psi_j(z)\exp(-ik_{nx}x),$$

$$\Phi^{(2)}(x, z) = \sum_n C_n\psi_n(z)\exp(ik_{nx}x).$$

Here, $B_j$ and $C_n$ are the constant coefficients determining the amplitudes of the waves reflected in the quantum wells in subbands $E_j$ and passed into $QW_2$ in the subbands $E_n$; $k_j$ and $k_n$ are the wave numbers corresponding to the particle motion along the $x$ axis in these regions; $k_j = [2m^*(E - E_j - E_y)]^{1/2}/\hbar$; and $k_n = [2m^*(E - E_y - E_n)]^{1/2}/\hbar$. The reflection and transformation of elec-