INTRODUCTION

The use of the apparatus of conformal transformations is quite efficient for the solution of 2D problems in the quasi-static approximation [1–3]. Analytical expressions for the field are obtained from the rigorous solution to the electrostatic problem involving axially symmetric irregularities projecting from a half-space. The effect function of an irregularity is introduced to extend the quasi-statics results to the far field. The effect of the irregularity on the far field is determined with the help of the effect function asymptotic found from the near field. With the use of the equivalent source dipole moment the containing the asymptotics, the Sommerfeld problem can be solved in the case when there is no irregularity and the lower half-space has arbitrary properties.

The described approach to the quasi-static solution of 2D problems with an axially symmetric irregularity is also developed for the problem of two bodies that are two semicylinders located on a perfectly conducting plane and that have arbitrary properties [4]. In [4], expressions for the electric field are obtained and the effect function is analyzed for an arbitrary distance between the semicylinders of various curvatures. It is found that the field observed near the surface of the semicylinder with a source grows as the distance between the semicylinders decreases. The reason for this growth is not revealed in [4] and will be discussed below for the problem under consideration.

In this study, we continue to investigate the problem of two bodies and solve the problem for an embedded semicylinder. We analyze the field of a filament of electric dipoles (EDs) that is placed near a hollow semicylinder placed on a perfectly conducting plane. The effect function is studied for various displacements of the cavity of an arbitrary dimension inside the semicylinder. We investigate the distribution of the induced charge density on the surface of the outer semicylinder and discuss the relationship between the outer semicylinder and the growth of the field near the semicylinder’s surface. The dipole moments of the sources located near the outer semicylinder or inside the cavity are assumed to be oriented orthogonally (a vertical ED (VED)) or in parallel (a horizontal ED (HED)) to the perfectly conducting plane.

1. THE POTENTIAL AND DENSITY OF CHARGES INDUCED ON THE SURFACE OF A SEMICYLINDER

First, consider the electrostatic problem for the field produced by a filament of point charges \( q_0 \) that is located on the top of one of semicylinders. After that, we consider the field of dipole sources. We assume that the axes of semicylinders with radii \( r_1 \) and \( r_2 \) and permittivities \( \varepsilon_1 \) and \( \varepsilon_2 \) are spaced by distance \( x_0 \) (Fig. 1a). According to the reflection theorem, the problem is reduced to determination of the field produced by two symmetrically located filaments of point charges in a medium with permittivity \( \varepsilon_0 \) that contains a cylinder with a displaced cylindrical cavity.

In the plane \( z = x + iy \) of the cross section of the cylinders, we apply the conformal transformation

\[
w = \frac{z - z_1}{z - z_2},
\]

where \( w = U + iV \) and \( z_1 \) and \( z_2 \) are points that belong to the line of the centers of both circles and that are symmetric with respect to them [5]: \( z_1 z_2 = r_1^2 \) an \( (z_1 - x_0)(z_2 - x_0) = r_2^2 \). In polar coordinates, relationship (1) between the variables \( w = R \exp(i\varphi) \) and
\( \mathbf{z} = r \exp(i\psi) \) has the following form with the notation \( \mathbf{z}_\pm = z_1 \pm z_2 \):

\[
R^2 = r^2 - 2z_1r \cos \psi + z_1^2, \\
\rho^2 = z_2^2R^2 - 2z_1z_2R \cos \varphi + z_1^2, \\
\tan \varphi = \frac{z_2r \sin \psi}{r^2 - z_2r \cos \psi + z_1}, \\
\tan \psi = \frac{z_2 \rho \sin \varphi}{z_2R^2 - z_1R \cos \varphi + z_1}.
\]

In terms of the parameters \( \gamma = r_1/r_2 \), \( (\gamma < 1) \) and \( \sin \alpha = \frac{x_0}{r_1(1-\gamma)} \), the positions of points \( z_1 \) and \( z_2 \) can be represented as

\[
z_{1,2} = \frac{r_1(1+\gamma)}{2\sin \alpha} \left[ 1 + \frac{1-\gamma}{1+\gamma} \sin^2 \alpha \pm \cos \alpha \sqrt{1 - \frac{(1-\gamma)^2}{(1+\gamma)^2} \sin^2 \alpha} \right].
\]

Here, parameter \( \alpha \) determines the distance between the axes of the semicylinders. This parameter ranges from a small positive value \( \alpha > 0 \) at a very short distance between the semicylinders' axes \( (x_0 > 0) \) to \( \alpha = \pi/2 \) in the case when the semicylinders contact \( (x_0 = r_1 - r_2) \).

As a result of transformation (1), two eccentric circles in plane \( \mathbf{z} \) become concentric circles in plane \( \mathbf{w} \) (Fig. 1b) with the radii

\[
R_1 = \frac{r_1z_1}{r_1^2 - z_2}, \quad R_2 = \frac{r_2z_2}{r_2^2 - (x_0 - z_1)^2},
\]

Then, the region \( r < r_1, \sqrt{r^2 - 2rx_0 \cos \varphi + x_0^2} > r_2 \) between the two circles becomes the ring region \( R_2 > R > R_1 \), the inner circle \( \sqrt{r^2 - 2rx_0 \cos \varphi + x_0^2} < r_2 \) becomes the outer region \( R > R_1 \), and the outer region \( r > r_1 \) becomes the inner circle \( R < R_1 \). Using representation (3), let us express radii \( R_1 \) and \( R_2 \) (4) through parameters \( \alpha \) and \( \gamma \):

\[
\begin{bmatrix} R_1 \\ \gamma R_2 \end{bmatrix} = \frac{1 + \gamma}{2\sin \alpha} \left[ 1 \pm \frac{1-\gamma}{1+\gamma} \sin^2 \alpha + \cos \alpha \sqrt{1 - \frac{(1-\gamma)^2}{(1+\gamma)^2} \sin^2 \alpha} \right].
\]

Note that, for small \( \alpha \), (5) implies

\[
\begin{align*}
R_1 &\approx \frac{1 + \gamma}{\sin \alpha} \left[ 1 - \frac{\gamma^2}{(1+\gamma)^2} \sin^2 \alpha \right], \\
R_2 &\approx \frac{1 + \gamma}{\gamma \sin \alpha} \left[ 1 - \frac{1}{(1+\gamma)^2} \sin^2 \alpha \right].
\end{align*}
\]

Therefore, as \( \alpha \to 0 \), we have the relationship \( R_1 > R_2 \gg 1 \) for the radii. In the other limit case, when the semicylinders contact, we have the parameter \( \alpha \to \pi/2 \), and radii (5) of the cylinders approach unity:

\[
R_1 \approx 1 + \sqrt{\gamma} \cos \alpha, \quad R_2 \approx 1 + \cos \alpha/\sqrt{\gamma}.
\]

In variables \( \mathbf{w} \), where \( \mathbf{z} - \mathbf{z}_i = \mathbf{z} - \frac{w - w_i}{(1-w)(1-w_i)} \), the potential \( \Phi^0 = -\frac{q_0}{2\pi \varepsilon_0} \ln |\mathbf{z} - \mathbf{z}_i| \) of the source field in the original infinite physical space takes according to (1)