The Electromagnetic Field of a Horizontal Antenna under the Interface between Two Media

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Abstract—Excitation of electromagnetic waves in a two-layer medium by a horizontal antenna or an antenna with flooded electrodes that is situated on the water surface is investigated. The region below the interface is often necessary. We use the two-layer model of a medium with a plane interface to simplify calculations. The main attention is given to finding formulas that are valid for the whole lower half-space including the neighborhood of a source. Consider also the limit passage to the distance exceeding the thickness of several skin layers from a source.

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INTRODUCTION

Consider the structure of the field formed by a linear horizontal antenna, which is grounded or situated on the water surface and has flooded electrodes. In practical applications (communication, operation, geological exploration), the information about the character of the field change in the region under the interface is often necessary. We use the two-layer model of a medium with a plane interface to simplify calculations. The main attention is given to finding formulas that are valid for the whole lower half-space including the neighborhood of a source. Consider also the limit passage to the distance exceeding the thickness of several skin layers of the conducting medium. We rely on the classic approach used in works \cite{1,2} for calculating the medium interface field, which is the main quantity applied in the geological exploration.

1. THE ELECTRIC VECTOR POTENTIAL. THE QUASI-STATIONARY APPROXIMATION

Consider the radiation of a grounded antenna with flooded electrodes in a two-layer medium. The antenna has the length $2L$ and is fed by the current with the harmonic dependence on time $\exp(-i\omega t)$. The system of coordinates is chosen as follows. The center of Cartesian coordinates is placed in the antenna center $\rho$, and the distance from an arbitrary antenna point $\rho \eta$. We consider that the medium in the region $z > 0$ is practically nonconducting (i.e., $\sigma = 0$ and the presence of $+$ at the zero indicates a small absorption) with the permittivity $\varepsilon_0 = 10^{-9}/36\pi$ F/m and the permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m. We suppose that the electromagnetic parameters of the region $z < 0$ are $\varepsilon$, $\mu_0$, and $\sigma_0$. The problem of excitation of the electromagnetic field by exterior current $\vec{J}$ is reduced to the solution of the Helmholtz equations for electric vector potential $\vec{A}$ with the corresponding boundary conditions \cite{2–4}. Since we consider the radiation of monochromatic waves, it is convenient hereinafter to use the equations for complex amplitudes corresponding to monochromatic components, i.e., $\vec{A} = \vec{A}_0 \exp(-i\omega t)$, $\vec{E} = \vec{E}_0 \exp(-i\omega t)$, and $\vec{H} = \vec{H}_0 \exp(-i\omega t)$, where $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields, respectively. Taking into account that the source is oriented along axis $x$ (Fig. 1), we can represent vector $\vec{A}$ in the form of two components

$$\vec{A}^{(j)} = A_x^{(j)} \hat{e}_x + A_z^{(j)} \hat{e}_z,$$

where the sign $j = 0, 1$ indicates the medium and $\hat{e}_x$ and $\hat{e}_z$ are the unit vectors oriented along axes $x$ and $z$, respectively.
The subsequent step is the solution of the system of equations
\[ \nabla^2 \mathbf{A}^{(j)} + k_j^2 \mathbf{A}^{(j)} = -\mathbf{J}_\text{ext}, \quad j = 0, 1 \]
with the boundary conditions
\[ \mathbf{A}^{(0)}(z=0) = \mathbf{A}^{(1)}(z=0), \quad \frac{\partial \mathbf{A}^{(0)}}{\partial z}(z=0) = \frac{\partial \mathbf{A}^{(1)}}{\partial z}(z=0), \]
\[ \frac{1}{k_0^2} \text{div} \mathbf{A}^{(0)}(z=0) = \frac{1}{k_1^2} \text{div} \mathbf{A}^{(1)}(z=0). \]

Wave numbers \( k_0 \) and \( k_1 \) entering the system of Eqs. (1) and (2) are determined by the expressions
\[ k_0 = \frac{\omega}{c}(1 + i\varepsilon_0) \quad \text{and} \quad k_1 = \frac{\omega}{c} \sqrt{\varepsilon_1 + i\frac{\sigma_1}{\omega\varepsilon_0}} = \frac{\omega}{c} \sqrt{\varepsilon_1}. \]

Let us find the solution to system (1) with boundary conditions (2) for a point grounded (flooded) horizontal source located at the coordinate origin. In this case,
\[ J^{(0)} = J \Delta \hat{\delta}(x) \hat{\delta}(y) \hat{\delta}(z-0) \hat{\varepsilon}_x, \]
\[ J^{(1)} = J \Delta \hat{\delta}(x) \hat{\delta}(y) \hat{\delta}(z+0) \hat{\varepsilon}_x, \]
where \( \hat{\delta} \) is the delta function, \( J \) is the current, \( \Delta \) is the dipole moment and \( \Delta \) is the dipole length tending to an infinitely small quantity.

The transition to the fields corresponding to the excitation by a linear antenna of length \( 2L \) is realized as the result of summation over the antenna length. In particular, electric field \( \mathbf{E}^{(j)} \) excited by a linear antenna is
\[ \mathbf{E}^{(j)} = \sum \mathbf{E}^{(j)}(\rho, z) = \int \mathbf{E}^{(j)}(\rho, z) \rho d\eta, \]
where \( \mathbf{E}^{(j)}(\rho, z) \) is the field of the dipole located at point \( \eta \) of the antenna. The magnetic field satisfies the analogous relationship.

It is convenient to obtain the solution of system (1) with boundary conditions (2) in cylindrical coordinate system \((\rho, \varphi, z)\) in the form of the decomposition of the functions \( \cos (m/2)\varphi \), \( m = 0, 1, \ldots \), which form a complete system on the interval \([0, 2\pi]\) [6]. The further steps are well known [2, 5] and connected with the definition, taking into account the boundary conditions, and the corresponding functions, which enter the decomposition and depend on \( \rho \) and \( z \). Therefore, we omit the intermediate transformations and present the final result of calculation of \( A_x^{(j)} \) and \( A_z^{(j)} \) for source (3)
\[
A_x^{(j)} = J \Delta_x \frac{2\pi \exp\left(i\sqrt{k_0^2 - \lambda^2} |z| \right)}{4\pi \sqrt{k_0^2 - \lambda^2 + k_1^2 - \lambda^2}} J_0(\lambda \rho) \lambda d\lambda,
\]
\[
A_z^{(j)} = J \Delta_z \frac{\partial}{\partial \lambda} \left(k_0^2 - k_1^2\right) \times \int \frac{2\exp\left(i\sqrt{k_0^2 - \lambda^2} |z| \right)}{\sqrt{k_0^2 - \lambda^2 + k_1^2 - \lambda^2}} \left(k_0^2 \sqrt{k_0^2 - \lambda^2 + k_1^2 \sqrt{k_0^2 - \lambda^2}} \right) J_0(\lambda \rho) \lambda d\lambda,
\]
where \( J_0(\lambda \rho) \) is the Bessel function.

Fig. 1. Geometry of the problem.