INTRODUCTION

The problem of estimation of the states of dynamic systems by the observation data is one of the most important practical tasks. There exist different approaches to this problem that take into account the behavior description features of the dynamic system, the nature of the disturbances influencing its behavior, as well as the nature of the errors of measurements used for the state estimation. In practice, the realizability of the proposed algorithms subject to limited capacity of the computers and the necessity of real-time estimation of the quality of the state estimation are important factors influencing the choice of the approach to estimating the states of a dynamic system.

Thus, the estimation methods develop in several directions: approximation of attainability domains of a state of dynamic system using ellipsoids [1–6], robust estimating [7–22], estimation based on the probabilistic–guaranteeing approach [23–26] including the notion of the guaranteed estimation used in one sense or another. The methods of optimal and suboptimal estimation using the Kalman–type filters have also become widespread for solving applied problems of stochastic describing of dynamic systems behavior due to ease of realization of these methods considering the limited capacity of computers. The application of the Kalman filters is described for both linear [27–29] and nonlinear [30, 31] formulation of the estimation problem, as well as for the case of uncertainties in the model description [31, 32].

However, note that realization of Kalman–type suboptimal filters implies the problem of the quality of the performed estimation, because, unlike the optimal Kalman filter, the matrix calculated in the covariance channel of the suboptimal filter is not any more the covariance matrix of the error of the state vector estimation.

This paper addresses the synthesis of the suboptimal reduced Kalman filter with guaranteed estimation quality for description of the state of dynamic system and the process of measurement by linear stochastic equations with known parameters. Optimal solution of the estimation problem in this formulation can be obtained using the Kalman filter with the state vector including all non-white disturbances. However, in practice, the limited capacity of computers in the presence of a large number of disturbances with complicated description admits realization of suboptimal reduced Kalman filter only. The state vector of such a filter is less than the state vector of an optimal filter and does not include, e.g., colored disturbances with small correlation intervals.

There exist two approaches to synthesis of reduced filters. The first approach chooses parameters of the reduced filter using a computational model with state vector of a smaller dimension that approximates the model describing the real behavior of the dynamic system [30–32]. In the second approach, the synthesis of the reduced filter is based on the full model describing behavior of the dynamic system; parameters of the reduced filter are determined on the base of solving the minimization problem for the trace of the real covariance matrix of the estimation error given by linear combination of components of the state vector [33–37]. Note that the first suboptimal approach to the reduced filter synthesis implies the problem of the quality estimation for the obtained estimate. The second approach includes different computational procedures.
for solving the minimization problem for the trace of the real covariance matrix of the estimation error [38–40]. However, realization of these procedures requires a large amount of computations, which exceeds the amount of computations necessary for realization of the optimal Kalman filter. This reduces considerably the practical value of this approach.

This paper suggests using the guaranteed estimation principle for the synthesis of reduced Kalman filter. This principle consists in choosing the filter parameters in such a way that the solution of the Riccati equation, the matrix calculated in the covariance channel of the reduced filter, is larger (in the sense of quadratic forms inequality) than the real covariance matrix of the estimation error produced by this filter. In this case, this matrix can be considered as a guaranteed accuracy measure for the performed suboptimal estimation of the state of dynamic system. Such an approach makes it possible to simplify the procedure of synthesis of a reduced filter providing estimation with guaranteed accuracy and may be useful for solving different applied problems.

1. FORMULATION OF THE SYNTHESIS PROBLEM FOR THE REDUCED FILTER WITH GUARANTEED ESTIMATION QUALITY

Let us consider the equation describing behavior of a dynamic system subject to disturbances of non-white noise character in the following form:

\[
\dot{X} = \begin{bmatrix} \dot{X}_0 \\ \dot{X}_1 \end{bmatrix} = \begin{bmatrix} F_0 & F_1 \\ 0 & F \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \end{bmatrix} + \begin{bmatrix} \xi_0 \\ \xi_1 \end{bmatrix} = FX + \xi, \quad (1.1)
\]

where \(X(t)\) is the state vector of the system; \(X_0(t)\) is the state vector of the forming filter used for description of the of non-white noise disturbances that are not supposed to be specified in the produced reduced filter by the results of the current measurements; \(\xi(t)\) is the vector of white noise disturbances of intensity \(Q(t)\) following the normal distribution: \(\xi(t) \sim \mathcal{N}(0, Q(t))\); and \(X(0)\) is the initial state vector following the normal distribution: \(X(0) \in \mathcal{N}\{\bar{X}, D(0)\}\). The matrices \(Q(t)\) and \(D(0)\) have the following form:

\[
Q = \begin{bmatrix} Q_0 & 0 \\ 0 & Q_1 \end{bmatrix}, \quad D(0) = \begin{bmatrix} D_0(0) & D_1(0) \\ D_0^T(0) & D_1(0) \end{bmatrix}.
\]

We will also assume that the performed measurements are described by the equation

\[
Z(t) = HX(t) + \nu(t) = HX(t) + \nu(t), \quad (1.2)
\]

where \(\nu(t)\) is the vector of the white noise errors of the measurements of intensity \(R(t)\) distributed according to the normal law; \(\nu(t) \in \mathcal{N}\{0, R(t)\}\). The processes \(\xi(t)\) and \(\nu(t)\) are independent.

The matrices \(F, H\) in the expressions (1.1), (1.2) are assumed to be non-stationary, and the matrix \(R(t)\) is nonsingular. For short, the argument \(t\) in matrix expressions is omitted, when the dependency of parameters on this argument is obvious.

It is known [27, 28] that if estimation of the state vector \(X(t)\) is based on the procedure described by the equation

\[
\dot{X} = FX + K(Z - HX),
\]

where \(K(t)\) is a coefficient of the filter gain, then the real covariance matrix of the estimation error of the state vector \(X(t)\) can be determined from the equation

\[
\dot{P} = (F - KH)D + D(F - KH)^T + KRK^T + Q. \quad (1.3)
\]

Since the state vector \(X(t)\) is not estimated based on the measurements \(Z(t)\) in the synthesized reduced filter, the filter gain \(K(t)\) has the following form:

\[
K(t) = \begin{bmatrix} K_0 \\ 0 \end{bmatrix},
\]

where the zero block corresponds to the vector \(X_0(t)\). Using block representation of the matrices involved in (1.3), the equation of the real covariance matrix of the estimation error of the state vector \(X(t)\) with the measurement matrix \(H = [H_0, 0]\) can be expressed as:

\[
\begin{bmatrix} \dot{D}_0 & \dot{D}_1 \\ \dot{D}_0^T & \dot{D}_1^T \end{bmatrix} = \begin{bmatrix} F_0 & F_1^T & D_0 & D_1 & F_0^T \\ 0 & F & D_0^T & D_1 & F_0^T \\ D_0 & D_1 & F_0^T & F_0 & F_0^T \\ D_1 & D_1 & F_0^T & F_0 & F_0^T \\ 0 & 0 & F_0 & F & F_0^T \\ 0 & 0 & 0 & 0 & F \end{bmatrix} + \begin{bmatrix} K_0 & 0 \\ -K_0 & 0 \end{bmatrix} R, \quad (1.4)
\]

where the block \(D_0(t)\) corresponds to the vector \(X_0(t)\), \(D_0(t) = F_0 - K_0(t)H_0\). The following expressions for the blocks \(D_0(t), D_1(t), D_0(t)\) of the real covariance matrix \(D(t)\) can be obtained from this equation:

\[
\begin{align*}
\dot{D}_0 &= \tilde{F}_0D_0 + D_0\tilde{F}_0^T + F_2D_1^T + D_2F_1^T + K_0R_{K_0^T} + Q_0, \\
\dot{D}_1 &= \tilde{F}_1D_1 + D_1\tilde{F}_1^T + F_2D_1 + Q_1, \\
\dot{D}_0^T &= \tilde{F}_0D_0^T + D_0\tilde{F}_0^T + F_2D_1^T + D_2F_1^T + K_0R_{K_0^T} + Q_0.
\end{align*}
\]

The problem of synthesis of the reduced filter can be formulated as follows, using a filter of the form:

\[
\begin{align*}
\dot{X}_0 &= F_0\dot{X}_0 + K_0^*(Z - H_0\dot{X}_0), \\
K_0^* &= P^*H_0^TR^{-1}, \\
\dot{P}^* &= F_0P^* + P^*F_0^T - P^*H_0^TR^{-1}H_0P^* + Q^*, \\
P^*(0) &= P_0^*.
\end{align*}
\]

where the filter parameters \(Q^*(t)\) and \(P_0^*\) are chosen in such a way that for any \(t\) the following inequality is valid: \(D_0(t) \leq P^*(t)\). Assume that in the sense of this inequality such filter provides guaranteed quality for estimation of the state vector \(X_0(t)\). Here and in what follows, the matrix inequality is meant as an inequality of quadratic forms.