INTRODUCTION

When a system (plant) is controlled under parametric uncertainty and only its scalar input and output are available for measurement, it is one of fundamental assumptions that said system is a minimum-phase one. A linear system (plant) is referred to as a minimum-phase one if its transfer function has a Hurwitz numerator, and non-minimum-phase, if its transfer function has a non-Hurwitz numerator [1]. In design of adaptive and robust control schemes, the requirement for the system to be a minimum-phase one is explained by the desire to design stable controllers [1].

Nowadays, only a small number of solutions have been proposed for the control problem for non-minimum phase systems with parametric uncertainty and scalar input and output. For instance, shunting is used for this class of plants in [2]. However, this method is only applicable to stable linear plants. In addition, shunting turns out to be ineffective under disturbances, because the regulation is driven by the augmented signal, which is a sum of the outputs of the plant and the shunt. A series compensator that makes it possible to obtain a new augmented model of the plant under vector control and thus compensating for positive zeros of the transfer function of the plant was independently proposed in [3, 4]. However, the algorithm from [3, 4] is only effective for the stabilization of non-minimum phase systems not subjected to uncontrollable external disturbances.

The control of non-minimum phase systems becomes significantly more complex when a single plant is replaced with a set of interrelated plants. Dynamical network will be interpreted as a set of dynamical subsystems (nodes) connected by physical and information connections. Non-minimum phase (minimum-phase) dynamical networks will be interpreted as dynamical networks, whose subsystems are non-minimum-phase (minimum-phase).

It is a current trend that dynamic networks come into universal use. Numerous examples of this trend include multiprocessor systems for data transmission and processing, various transportation networks, advanced manufacturing networks, complex crystal structures, nanostructure objects, etc. Particularly, non-minimum phase dynamic systems are represented by coordinated control systems for aerial, underwater, and robotic vehicles, distributed control systems for electric power networks, and etc.

Nowadays, nearly the whole range of available methods and approaches designed for controlling a single plant has been applied to dynamical systems. For instance, the problem of coordinate-based cooperative control of a dynamic network was considered in [5] taking into account the network topology, robustness properties of the network were studied, and a generalization of the Nyquist criterion for network systems was proposed. In [6], modal control was used to solve the synchronization problem for a network of plants with measured vector input and output under the assumption that the graph associated with the network has a spanning tree. The necessary and sufficient synchronization conditions, which impose con-
The algorithm designed for minimum-phase systems remains efficient for non-minimum phase systems. Then, the solution for a network without a master and with delays in communication channels is considered. The auxiliary circuit technique, which was first proposed in [10, 11] and generalized for control of a network, is used. The present paper is dedicated to solving the said problem.

In this paper, we consider the solution of the robust control problem for a specific class of non-minimum phase dynamical networks under parametric uncertainty and uncontrollable external disturbances. The solution is constructed under the condition that only scalar input and output signals are measured in each local subsystem of the network. To take into account the topology of the network, the digraph whose vertices are associated with the corresponding network nodes, and arcs are associated with information connections between the corresponding subsystems of the network is introduced. To construct a control law, the auxiliary circuit technique, which was first proposed in [10, 11] and generalized for control of a dynamic network in [12–14], is used. First, the solution of the problem is considered for a network with the master subsystem that determines the desired behavior of the other local subsystems of the network. Then, the solution for a network without a master and with delays in communication channels is considered. The designed control systems ensure the synchronization of the network with the required accuracy. The conditions depending on the parameters of the network and the control system under which the algorithm designed for minimum-phase systems remains efficient for non-minimum phase systems are obtained. Numerical examples are presented to illustrate the efficiency of the algorithm.

1. STATEMENT OF THE CONTROL PROBLEM FOR A DYNAMIC NETWORK WITH A MASTER

Consider the dynamic network $\mathcal{S}$ consisting of $k$ subsystems $S_i$ ($i = 1, \ldots, k$) and the master subsystem $S_L$. The goal of control (synchronization) for a dynamic network with the master is to find controllers that ensure that the decision of each local subsystem is close to the decision of the master [6, 9, 13].

Define the digraph $\Gamma = (V, E)$ associated with the network $\mathcal{S}$. The vertices in $\Gamma$ are associated with the corresponding subsystems $S_i$ and the master $S_L$. $V = \{v_1, \ldots, v_k, v_L\}$ is the set of vertices, and $E \subseteq V \times V$ is the set of arcs. Let $C = (c_{ij})$ and $S = (s_{ij})$ be the adjacency matrices of $\Gamma$ such that $c_{ij} = 1$ and $s_{ij} = 1$ if $j \in N_i$, and $c_{ij} = 0$ and $s_{ij} = 0$ otherwise. Here, $N_i = \{v_j \in V : (v_j, v_i) \in E\}$ is the set of adjacent vertices for the vertex $v_i$. The notation $(v_j, v_i), (v_i, v_j) \in E$ implies that the information is transmitted from subsystem $S_j$ to subsystem $S_i$ and from the master $S_L$ to $S_i$. In addition, we consider the digraph $\Gamma_0$ associated with the network $\mathcal{S}$, where the master $S_L$ is excluded.

Let the dynamic processes in each slave subsystem $S_i$ be described by the equation

$$Q_i(p)y_i(t) = k_iR_i(p)u_i(t) + f_i(t), \quad p^{-1}y_i(0) = y_{i0}, \quad i = 1, \ldots, k,$$

where $y_i(t) \in R$ is the controlled variable, $u_i(t) \in R$ is the control signal, $f_i(t) \in R$ is a smooth bounded uncontrollable external disturbance, $Q_i(p)$ and $R_i(p)$ are linear differential operators, $\deg Q_i(p) = n_i$, $\deg R_i(p) = m_i$, $m_i - n_i \geq 1$, $k_i > 0$, $p = d/dt$ is the time differentiation operator, and $y_{i0}$ indicates the unknown initial conditions.

The model of the master $S_L$ is described by the equation

$$Q_L(p)y_L(t) = k_Lr(t).$$

Here, $y_L(t) \in R$ is the measured output, $r(t) \in R$ is the bounded reference signal (input), $Q_L(p)$ and $k_L > 0$ are the known operator and coefficient respectively, and $\deg Q_L(p) = n$. A continuous control is to be synthesized that synchronizes the network $\mathcal{S}$ with the required accuracy $\delta > 0$; i.e., the desired inequality

$$|y_i(t) - y_L(t)| < \delta \quad \text{for} \quad t > T,$$

is fulfilled.