The Wave Field Patterns of the Propagation of Longitudinal and Transverse Elastic Waves in Grain-Oriented Rocks

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Received April 15, 2008

Abstract—In the work under consideration, on the basis of data analysis on the velocities of quasi-longitudinal $V_p$ and quasi-transverse $V_s$ waves—measured for rock samples at high pressures and temperatures, and analogous velocities, calculated on the basis of the grain orientation distribution function, reconstructed from the neutron-diffraction textural experiment—the indicative inconsistencies between the experimental and model characteristics were inferred. The theoretical analysis of the wave field patterns of the propagation of longitudinal and transverse elastic waves in the anisotropic media is carried out. It is established that, in the general case, in the anisotropic inhomogeneous media the velocities of $V_p$ and $V_s$ propagation, measured experimentally and obtained from the modeling, cannot coincide due to the existence of the physical coupling between the vibrations of two types: transverse and longitudinal vibrations.

PACS numbers: 91.30.Cd
DOI: 10.1134/S1069351309050061

INTRODUCTION

The anisotropy of rocks is caused by those conditions, which were set at different depths of the Earth’s lithosphere, by the mineral composition and by the rock structure, for example, by their crystallographic and mechanical textures [Crankin, 1985; Babuska and Cara, 1991]. Traditionally the rock anisotropy under atmospheric and high pressures was investigated based on the measurements in three mutually perpendicular directions for the samples in the form of a cube [Birch, 1960; 1961]. More complete information is given by the study of the velocity anisotropy of longitudinal waves $V_p$ for samples in the form of a globe, which makes it possible to measure the velocities in any direction both under atmospheric and high hydrostatic pressures [Pros, 1977].

Besides the measurements by Russian and foreign authors, calculations are carried out on the effective elastic characteristics of a medium for the purpose of modeling the propagation of $V_p$ and $V_s$ waves in inhomogeneous media. In the work [Kalinin and Bayuık, 1987] effective elastic modules are calculated for isotropic rocks. In the works [Shermergor et al., 1987; 1991; Walter et al., 1993; Kalinin and Bayuık, 1994; Mainprice and Humbert, 1994; Bayuık and Kalinin, 1997; Sobolev et al., 2001] the possibilities of theoretical methods are considered, the calculations of the effective elastic and piezoelectric constants are carried out on the basis of the experimentally established grain orientation distribution functions, and the values of the $V_p$ and $V_s$ wave velocities are also obtained for anisotropic rocks.

For over ten years, the authors [Ivankina et al., 1999; Nikitin et al., 2001a; 2001b] carried out the systematic study of the factors which determine the elastic anisotropy of rocks at different pressures and temperatures by acoustic methods in combination with the neutron diffraction textural analysis. The comparison of the results of the direct measurements of ultrasound velocity in the samples of biotite gneiss from the Kola Superdeep borehole SG -3 and the Outokumpu Deep Drill Hole (Finland) with the results of the modeling of the propagation of quasi-longitudinal and quasi-transverse waves on the basis of the neutron diffraction textural analysis [Ivankina et al., 2005; 2007] revealed the difference in the behavior of the velocities of quasi-longitudinal waves. It was noticed that the model values of the anisotropy coefficient of the velocities of quasi-longitudinal waves, calculated for the grain-oriented biotite gneiss, are lower than the values, obtained from direct ultrasonic measurements.

This difference is indicative of the fact that besides the crystallographic texture, there are other factors, which influence the anisotropy of quasi-longitudinal elastic waves in the rocks.

The theoretical examination of the interaction of an anisotropic (textured) medium with the elastic quasi-longitudinal and quasi-transverse waves propagated in it is the basic milestone of this work.
THE RESULTS OF LABORATORY MEASUREMENTS AND THEORETICAL MODELING OF THE VELOCITIES OF ELASTIC WAVES OF THE ROCK SAMPLES

In order to determine the seismic anisotropy and to estimate the contribution of the oriented cracks and crystalllographic texture of minerals in the bulk anisotropy of the rock samples two different methods were used [Ivankina et al., 2005]. First, the measurements of the velocities of \(P\) - and \(S\) - waves in three orthogonal directions as the functions of pressure and temperature were conducted and, second, the 3D distribution of the velocities of elastic waves was modeled on the basis of the neutron data on the sample textures. For a comparative study the borehole samples of biotite plagioclase gneisses (K8802, K9002) from the Archean basement of the Kola Superdeep borehole SG-3 and the sample of gneiss (OKU818) from the Outokumpu Deep Drill Hole (Finland) were selected. The numbers of the samples show the sampling depth in meters. The mineralogical composition of the samples is approximately identical, but the percentage of mineral phases in them differs. The sample from the SG-3 borehole (K9002) contains plagioclase (55%), quartz (26%), biotite (9%), muscovite (7%), and impurities (3%). The K8802 sample contains plagioclase (61%), quartz (27%), biotite (7%), muscovite (5%), and impurities (3%). The OKU818 sample contains plagioclase (36%), quartz (40%), biotite (22%), and impurities (2%). A distinctive characteristic of the OKU818 sample is its intense schistosity (foliation), formed by the platy arrangement of the foliated biotite minerals. In samples K8802 and K9002, the foliation is not so clearly expressed.

The modeling of the elastic properties of the samples during the first stage included the calculation of the mean elastic modules of each polycrystalline mineral fraction on the basis of the grain orientation distribution function, reconstructed from the neutron-diffraction data with the utilization of elastic modules of single crystals. The method of averaging the elastic modules used in the work for the polycrystalline grain-oriented aggregate ignores the presence of pores and cracks in the sample. The calculation was carried out according to the Vogt–Royce–Hill calculation scheme. For simplification of the problem the Vogt averaging was used, which, as is shown in practice by the calculations of the effective elastic modules [Seront et al., 1993], gives a satisfactory approximation of laboratory measurements. In the modeling of the velocity distribution of elastic waves, the volumetric content of each mineral phase in the sample was considered.

The laboratory measurements of the velocities of quasi-longitudinal (\(V_p\)) and quasi-transverse (\(V_s\)) waves were carried out with the samples of a cubic shape (with the edges 43 mm in length) in a pressure machine, with the use of the method of pulse ultrasonic sounding by the sensors, which have an operating frequency of 2 MHz. The instrumentation includes special equipment for the simultaneous recording of the velocities of longitudinal and orthogonally polarized transverse waves (\(S_L\), \(S_2\)) in three perpendicular directions. The measurements were conducted at pressures of up to 600 MPa and within a temperature range from room temperature to 600°C. A detailed description of the experimental method is contained in the work [Kern et al., 1997].

In the samples investigated from the Kola Superdeep borehole the plane of schistosity (foliation) was inclined with respect to the borehole sample axis. Since the borehole samples had a small diameter (60 mm) and small length, it was not possible to make cubes of the necessary size in accordance with the traditionally utilized structural system of coordinates \(X, Y,\) and \(Z\) (where \([Z]\) is perpendicular to the foliation, \([Y]\) is perpendicular to the foliation and lineation, and \([X]\) is parallel to the lineation). The system of sample coordinates \(A, B,\) and \(C\) was used, which was related to the axis of the borehole in such a way that \([C]\) was parallel to the axis of the borehole, and \([B]\) and \([A]\) were perpendicular to it; in this case the direction \([A]\) corresponds to the lineation. The radiating and receiving transducers were oriented in parallel and in such a way that the predominant particle displacements in \(S_L\) and \(S_2\) would be parallel and perpendicular to the \(AB, AC\), and \(BC\) planes, respectively. This arrangement differed from the subsequent experiment, when the sample from the Outokumpu Deep Drill Hole (Finland) was cut in the structural system of coordinates \(X, Y,\) and \(Z,\) and the measurements of elastic waves on it were conducted by the standard procedure [Kern et al., 2001], when \(P\)- and \(S\)-waves propagate in parallel and perpendicular to the foliation plane.

The basic pole figures of plagioclase, quartz, and biotite—calculated with the help of the grain orientation distribution function of the mineral phases, reconstructed from the results of the neutron-diffraction textural analysis—are presented in Figs. 1 and 2 for samples K8802 and K9002, correspondingly. The pole figures are the equal-area projections on the \(BC\) plane, where, for comparison purposes, the orientation of the axes of the structural system \(X, Y,\) and \(Z\) is also shown. Although biotite and muscovite enter into the composition of the K8802 sample, their textures were not reconstructed quantitatively because of the weakly developed texture and the poor statistics on the neutron-diffraction spectra. In the K9002 sample, the predominant orientation of biotite is noticeably more developed (Fig. 2). However, the OKU818 sample possesses the sharper biotite texture (Fig. 3), in which the percentage ratio of the biotite grains is higher in comparison with the gneisses from the Kola borehole. The preferred orientation of plagioclase and quartz of the studied biotite gneisses is also well expressed, but to a lesser extent.

The system of coordinates of the maps of velocities of \(P\)- and \(S\)-waves (Figs. 4, 5, and 6) coincides with the system of coordinates of the pole figures, which makes