INTRODUCTION

Until recently, a Newtonian fluid was used to model slow flows in the mantle. In this model, which describes diffusion creep, the deviatoric stress tensor is related to the deviatoric strain rate tensor by a linear law

$$\sigma_{ij} = 2\eta \dot{\varepsilon}_{ij}.$$  \hspace{1cm} (1)

The diffusion viscosity $\eta$ of polycrystalline material depends on temperature, pressure and grain size.

Nowadays, a non-Newtonian power-law fluid is usually assumed to model the mantle slow flows. This model adequately describes a steady-state dislocation creep which is observed in laboratory studies carried out at constant stresses and at temperatures and pressures approximating mantle conditions. The effective viscosity of a power-law model depends not only on the temperature and pressure, but also on the deviatoric stress. Rheology is determined by micromechanism which gives the minimal effective viscosity. For example, the estimates (Karato and Wu, 1993) show that the power-law dislocation creep dominates the upper mantle and the lower crust whereas the lower mantle is rather dominated by the diffusion creep.

To investigate geodynamical processes we must have a rheological model for the mantle which is valid when stresses change with time. The power-law fluid is not such a model. Besides, the power-law model does not take into account a transient creep observed in laboratory studies at small strains.

TRANSIENT CREEP OF ROCK

A typical experimental creep curve, which gives the dependence of creep strain on time at a constant stress applied at the initial moment, can be divided into three stages. At the first stage (transient creep), strain rate decreases (strain hardening). At the second stage (steady-state creep) strain rate is constant. At the third stage of the strain rate increases (strain softening) that associates with the formation and growth of microcracks leading to the destruction of the test sample rock. Laboratory studies show that the transition to the steady-state creep occurs at a certain strain and does not depend on the stress at which the experiment is carried out. The lower the constant applied stress, the longer the transient creep stage. The experiments also found that the transient creep strain is linearly dependent on the applied stress

$$2\varepsilon_{ij} = \sigma_{ij} f(t),$$  \hspace{1cm} (2)

where $f(t)$ is a creep function, and $t$ is a time. For mantle rocks at high temperatures the creep function is well described by the Andrade law

$$f(t) = t^m / A,$$  \hspace{1cm} (3)

where $A$ is an Andrade rheological parameter, and the typical value of exponent is $m = 1/3$. The Andrade law for transient creep has repeatedly confirmed in tests carried out at typical mantle pressures and temperatures (Berckhemer et al., 1979; Murrell, 1976).

Because Andrade law exponent less than unity, this law gives infinite strain rate and zero effective viscosity at the moment of stress application, which of course is not observed in tests. That’s why Jeffreys (1958) sug-
gested to use the modified Lomnits law instead of Andrade law. However, even at very short times (fra-
tions of a second) since the stress application, the laws of Jeffreys and Andrade are indistinguishable (Berk-
hemer et al., 1979).

At a constant steady-state creep strain rate depends nonlinearly on the applied stress

$$\dot{\varepsilon} = B\sigma^{-n}\sigma_i,$$  \hfill (4)

$$\dot{\varepsilon} = (\sigma_{kl}\sigma_{kl})^{-1/2}, \quad \sigma = (\sigma_{ij}\sigma_{ij} / 2)^{1/2},$$

where $\sigma$ and $\dot{\varepsilon}$ are the second invariants of deviatoric tensors of stress and strain rate, $B$ is a rheological parameter which characterizes the steady-state creep, a typical value of exponent is $n = 3$.

To generalize the results of creep tests at constant stress for stresses changing with time the Boltzmann linear theory can be used. This theory, valid for the case of sufficiently small strains, leads to the integral relation

$$2\varepsilon_{ij} = \int K(t)\sigma_{ij}(t-s)ds,$$  \hfill (5)

where $K(t)$ is an integral kernel of creep, which is determined by the creep function

$$K = \frac{df}{dt}.$$  \hfill (6)

As follows from (3) and (6), the kernel of creep, corresponding to the Andrade law, has the form

$$K(t) = mt^{m-1}/A.$$  \hfill (7)

Rheological model, which is described by equations (5) and (7), is called a model of Andrade. This model generalizes the Andrade law for the case of variable stresses. Equations (5) and (7), describing the model of Andrade, can be rewritten as

$$\sigma_{ij} = 2\int R(s)\varepsilon_{ij}(t-s)ds,$$  \hfill (8)

where $R(s)$ is the relaxation kernel

$$R(s) = As^{m-1}/\Gamma(m)\Gamma(1 - m),$$  \hfill (9)

and $\Gamma(m)$ is the gamma function.

**NONLINEAR INTEGRAL MODEL OF CREEP**

Much more difficult to generalize the rheological relation (4) describing steady-state creep for the case of time-dependent stresses because the rheology of rocks under variable stresses and large strains has not been investigated experimentally. The simplest generalization is the assumption that equation (4) holds for time-dependent stresses. The model described by equation (4) at any depending of stress on time is called the power-law non-Newtonian fluid. This model is commonly used in the study of thermal convection in the mantle, but in the case of non-stationary convection, its applicability is questionable. The power-law model is in good agreement with experiments at constant stress but this does not imply that this model is applicable under variable stress. In the power-law fluids, as in the Newtonian model (1), the current stress is determined by the current strain rate, i.e. by strains that exist in an infinitely short period of time before the current moment. In the real material, the current stress depends on the entire history of strains. In other words, the real material has a memory, in contrast to the power-law fluid.

Rheological models with memory are widely used in mechanics. The theory of simple fluids with fading memory is a fundamental rheological theory in modern continuum mechanics (Astarita and Marrucci, 1978). Decaying memory means that the current stress depends on the recent strains much stronger than on the strains that existed in the distant past.

Birger (1998) proposed a new nonlinear integral (having a memory) model for high-temperature dislocation creep of rocks. This model is consistent with the theory of simple fluids with fading memory and is determined by the equation

$$\sigma_{ij} = 2\int R(s)g(\varepsilon)\varepsilon_{ij}(t-s)ds,$$  \hfill (10)

where

$$g(\varepsilon) = 1, \quad \varepsilon \leq \varepsilon_{tr},$$  \hfill (11)

$$g(\varepsilon) = 0, \quad \varepsilon > \varepsilon_{tr},$$

$\varepsilon = (\sigma_{kl}\sigma_{kl})^{1/2}$ is the second invariant of the strain tensor, and $R(s)$ is defined by (9). Thus, equation (10) generalizes equation (8), corresponding to the Andrade model, for the case of large strains measured from the state at the time of observation. At this moment ($s = 0$), the strain $\varepsilon_s(t)$ is zero by definition. As follows from (10) and (11), the memory is cut off when the second invariant of the strain tensor exceeds the transition value $\varepsilon_{tr}$, which can be determined in experiments carried out at constant stress. Nonlinear integral model (10) reduces to the linear Andrade model (8) for flows associated with small strains. At stationary flows, causing large strains, the model is reduced, as shown in (Birger, 1998), to the model of power-law fluid with rheological parameters $B$ and $n$, whose values are determined by the rheological parameters $A, m,$ and $\varepsilon_{tr}$ in (10), (11). If $m = 1/3$ in the Andrade law, the relations are

$$n = 3, \quad 1/B \approx \frac{1}{3} \varepsilon_{tr} A^3.$$  \hfill (12)

The transition value of strain is estimated as $\varepsilon_{tr} \approx 10^{-3}$–$10^{-2}$. Note that the experimental measurement of the rheological parameters have large uncertainties (Korenaga and Karato, 2008), and therefore the used...