Abstract—We discuss the interpretation of the three-dimensional $\mathcal{N} = 8$ superconformal Chern–Simons matter theory with the gauge group of volume preserving diffeomorphisms as a model describing a six-dimensional self-dual gauge field coupled to scalars and spinors and its possible relation to the M5-brane.

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1. MOTIVATION

Since 2008 there has been a rather intensive activity in the study of three-dimensional superconformal theories describing the interaction of Chern–Simons gauge fields with matter supermultiplets. This activity was inspired by the papers by Bagger and Lambert [1, 2, 3], Gustavsson [4] and by Aharony et al. (ABJM) [5] which made a break through in the construction of $d = 3$ conformal gauge theories with $\mathcal{N} = 8$ and $\mathcal{N} = 6$ supersymmetries. By now the dust around this activity seems to settle down and one may pause and calmly overview the developments of this subject.

One of the main motivations behind this activity comes from the AdS4/CFT3 correspondence, which involves M-theory and, in particular, multiple M2-branes. The hope is that in this way one can make progress in understanding the M-brane theory as a possible microscopic formulation of M-theory, as well as to get a deeper insight into the structure of type IIA string theory compactified to AdS5 backgrounds.

One of the first persons who addressed the problem of understanding the effective world-volume theory of multiple M2-branes from this perspective was John Schwarz. In the paper [6] of 2004 he formulated main properties of the theory of multiple M2-branes and tried to construct it. The main features of the theory of $N$ M2-branes conjectured by J. Schwarz are:

- This should be a 3d gauge theory with $\mathcal{N} = 8$ linearly realized supersymmetries and the superconformal symmetry $OSp(8|4)$. The argument is based on the fact that $D = 11$ supergravity has the maximally supersymmetric solution with the geometry of $AdS_4 \times S^7$ supported by a non-zero flux of the 4-form gauge field strength. The isometry group of this solution is $OSp(8|4)$. In a dual picture this solution arises as a large-$N$ (or near horizon) limit of a stack of $N$ parallel M2-branes in (orbifolded) flat $D = 11$ space-time, that by the Maldacena’s AdS/CFT correspondence conjecture is described by a maximally supersymmetric superconformal theory.
- The R-symmetry of the $\mathcal{N} = 8$ supersymmetric theory should be $USp(8)$ since a single M2-brane has 8 scalar modes in a vector representation of $USp(8)$ which correspond to 8 directions in $D = 11$ transversal to the M2-brane worldvolume.
- The gauge group of the theory should include $U(N)$, with $N$ corresponding to the number of M2-branes or, in the dual picture, to $N$ units of magnetic flux through the 7-sphere. This argument has not found a direct evidence in $D = 11$ M-theory yet, but comes from the observation that M-theory is a strong coupling limit of type IIA string theory and, correspondingly, the M2-brane theory is a strong coupling limit of a low-energy effective worldvolume theory of $N$ coincident D2-branes. The latter is known to be a maximally supersymmetric $U(N)$ Yang–Mills theory in three dimensions. This theory is not conformal since the 3d YM coupling is dimensionful, but it is believed to have a strong-coupling (or, equivalently, infrared) limit in which the theory becomes conformal and describes the collection of M2-branes in $D = 11$. Further analysis has shown that the gauge group of $N$ coincident M2-branes is, actually, $U(N) \times U(N)$.
- The physical content of the theory of $N$ coincident M2-branes should comprise 8 scalars in fundamental representation of the gauge group as well as their superpartners, 8 Majorana spinor fields. The gauge field, if present, cannot be dynamical, because of the conformal invariance and supersymmetry. Hence it should be described by the conformally invariant Chern–Simons action which does not bring physical degrees of freedom.
- Finally, the M2-brane theory should be invariant under parity transformations, since its D2-brane counterpart is parity invariant. As was pointed out by J. Schwarz, this requirement (at least naively) is in contradiction with the assumption that the gauge field

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should be of the Chern–Simons type, since the latter violates parity.

Independently of the problems regarding the formulation of the M2-brane theory, the construction of maximally supersymmetric and conformally invariant \( d = 3 \) theories describing Chern–Simons gauge fields and their interaction with matter super multiplets are interesting problems per se, and they were approached in several papers [7–14, 6]. The net result was the construction of Chern–Simons models with up to \( \mathcal{N} = 3 \) supersymmetries. The break through in the construction of \( \mathcal{N} = 8 \) Chern–Simons–matter models has been made only relatively recently in [1–4] and [5]. Less supersymmetric \( \mathcal{N} = 4 \) theories have been constructed in [15, 16]. The BLG and ABJM constructions (and their subsequent, analysis) solved, in particular, the problem, of parity conservation and identified the structure of the gauge symmetry of the models which are intended to describe multiple M2-branes in certain \( D = 11 \) backgrounds.

An interesting outcome of the BLG construction is that it has brought to the attention of a wide community of theoretical and mathematical physicists a new gauge symmetry structure based on the so called 3-algebra that generalizes the notion of the Lie algebra. The 3-algebras, and, in general \( n \)-algebras were introduced by V. Filippov [17]. They are intimately related to the Nambu bracket [18] whose algebraic structure is that of an “\( \infty \)-algebra”.

Another aspect of the BLG model which has been studied rather intensively is its possible interpretation as an effective description of the dynamics of a single M-theory five-brane, rather than of multiple membranes. This relation has been studied from different perspectives. In this contribution we discuss a proposal put forward in [19–21] and further developed in [22]. It is based on the version of the BLG model in which the gauge symmetry is promoted to an infinite-dimensional group of so called volume preserving diffeomorphisms. We shall show that indeed, somewhat surprisingly, the BLG model, which is a three-dimensional theory of scalar and spinor fields interacting with gauge fields associated with the group of volume-preserving diffeomorphisms, can be reinterpreted as an effective six-dimensional theory whose physical content is the same as that of an M5-brane propagating in a certain eleven-dimensional super background. A related construction based on a mass-deformed BLG model was considered in [23].

2. \( \mathcal{N} = 8 \) SUPERCONFORMAL
CHERN–SIMONS–MATTER THEORY

Let us start, with a brief review of main properties of the Bagger–Lambert–Gustavsson model. It is a 3d theory which is conformally invariant and \( \mathcal{N} = 8 \) supersymmetric. Therefore, the superconformal symmetry of the BLG model is the supergroup \( OSp(8/4) \).

The model contains eight bosonic fields \( X^{a}(x^{\alpha}) \) \( (I = 1, \ldots, 8; a = 0, 1, 2) \) and 16 fermionic (3d Majorana spinor) fields \( \Psi_{a}(x) \) taking values in an \( n \)-dimensional (fundamental) representation of a gauge algebra \( g \). They interact with a Chern–Simons gauge field \( A_{a}(x) \) \( (a = 0, 1, 2) \) valued in the adjoint representation of \( g \). Therefore, the matter fields carry the index \( A = 1, \ldots, n \) of the fundamental representation, \( X^{a}_{A}(x) \), \( \Psi_{a}(x) \), and the gauge field carries a couple of indices \( A, B \) of the adjoint representation, namely \( A_{a}^{AB}(x) \).

The spinor (16-component) index of \( \Psi_{a}(x) \) is implicit. As soon as the construction should be related to the description of M2-branes in eleven dimensions, it is convenient to regard \( \Psi_{a}(x) \) as 32-component spinors subject to the \( \kappa \)-symmetry constraint which singles out 16 independent components

\[
\Gamma^{a}_{\beta} \Gamma^{b} \Psi = -\Psi, \quad (2.1)
\]

where \( \Gamma^{a} \) \( (a = 0, 1, 2) \) are 32 \times 32-component gamma-matrices along the worldvolume directions of the 3d theory. Together with eight \( \Gamma^{I} \) (transverse to the worldvolume) they form a complete set of \( D = 11 \) gamma matrices.

The model has eight \( g \)-valued scalar degrees of freedom and eight \( g \)-valued fermionic ones forming an \( \mathcal{N} = 8 \) supermultiplet with the linearized \( \mathcal{N} = 8 \) super-symmetry transformations having the form

\[
\delta X^{a} = i \epsilon \Gamma^{a} \Psi, \quad \delta \Psi = \slashed{D} X^{a} \Gamma^{a} \epsilon \epsilon + O(X^{3}, \epsilon), \quad (2.2)
\]

where \( \epsilon \) is the 16-component supersymmetry parameter, \( \Gamma^{a} \) and \( \Gamma^{I} \) are 11-dimensional gamma-matrices and \( \slashed{D} = \partial_{a} + A_{a} \) is the gauge-covariant derivative. The Chern–Simons gauge fields do not carry any physical degrees of freedom. The physical content of the BLG model and its symmetries are similar to those of a certain number of coincident membranes propagating in 11-dimensional superspace. This matching of the physical spectra and the symmetries is the basis of the conjecture that the BLG model provides us with an effective worldvolume description of coincident membranes in 11-dimensional M-theory.

Further analysis of the BLG model actually showed that the requirement of \( \mathcal{N} = 8 \) supersymmetry of its action drastically constrains the choice of the gauge groups. Actually, there are essentially only two options:

(i) The gauge group of local symmetries is \( SO(4) \sim SU(2) \times SU(2) \) or \( SU(2) \times U(2) \). So the gauge-symmetry index \( A \) takes four values. In this case the model describes two coincident membranes [27];

(ii) The possible choice of the gauge symmetry becomes much wider if one renounces positive definiteness of the 3-algebra quadratic form [24–26].