
PHYSICS OF ELEMENTARY PARTICLES
AND ATOMIC NUCLEI. THEORY

The Survey of Proton Structure Function with the AdS/QCD Correspondence¹

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Received May 18, 2018

Abstract—The proton structure functions at small x region in the holographic QCD are determined. In this study, the complicated nonperturbative interaction between the virtual photon and the target nucleon is described via the Pomeron swap, which corresponds to the reggeized graviton swap in the AdS space. We will show that the proton structure functions determined are consistent with the experimental data measured at HERA.

Keywords: deep inelastic scattering, Pomeron, Holographic QCD, Gauge/string correspondence

DOI: 10.1134/S154747711806002X

1. INTRODUCTION

The AdS/CFT correspondence provides a powerful tool for extra-dimensional model building. Qualitative features of electroweak symmetry breaking models with warped extra dimensions can often be predicted by analogy with the AdS/CFT correspondence. Even quantitatively, simple extra-dimensional models of QCD motivated by the AdS/CFT correspondence have proven successful at reproducing low-energy hadronic data like meson masses, decay constants, and coefficients of the chiral Lagrangian. These models fall into two classes: top-down models based on brane constructions in string theory, and bottom-up models which are more phenomenological. Both top-down and bottom-up models in this framework are referred to as AdS/QCD models or holographic QCD. In top-down AdS/QCD models, a brane construction in string theory is engineered which at low energies describes a gauge theory with features similar to QCD. In bottom-up AdS/QCD models, we specify an extra-dimensional spacetime geometry and the fields that propagate in them based on the properties of QCD which we would like to be incorporated. Boundary conditions on gauge fields break the higher-dimensional gauge invariance, while the corresponding global symmetry remains in the effective $3 + 1$ dimensional theory. This is the basic scenario in some basic bottom-up models. Rather than use scalar fields to break the chiral symmetry, modified spacetime geometry and boundary conditions can do the same. A more general approach includes the scalar fields with modified boundary conditions. The summary of our discussion thus far is

that AdS/QCD models seem to be reasonably reliable for predicting observables at below a few GeV, but tend to make poor predictions at higher energies. Until we better understand why certain models work especially well, we should at best trust only those predictions which are independent of model details such as the choice of spacetime geometry or boundary conditions. According to the AdS/CFT correspondence, finite temperature physics can be studied by introducing a black hole into the higher-dimensional spacetime. If the geometry is allowed to vary in a certain class of AdS/QCD models, it seems that the AdS geometry provides an especially good fit to data, but certain predictions are relatively insensitive to the details of the choice of geometry. The benefit of an extra-dimensional approach is that several features of QCD are immediate consequences of extra dimensions. QCD sum rules, vector meson dominance, and hidden local symmetry are all natural features of extra-dimensional models.

At small Bjorken- x region the structure functions of hadrons supply unique opportunities to understanding quark-gluon structure of QCD that is one of the most important subjects in high energy physics in last years. With the lepton-nucleon deep inelastic scattering (DIS) one can evaluate the structure functions. The structure functions are stated by this two kinematical variables, the Bjorken scaling variable x and the photon four-momentum squared Q^2 . These kinematical variables effect dynamics of quarks and gluons in a nucleon. When x is not small ($x > 0.01$) and Q^2 is large enough ($Q^2 > 1 \text{ GeV}^2$), perturbative QCD approaches are available and with respect to the initial conditions of parton distribution functions, one can predict the cross sections. If x is not small but Q^2 is small, we must consider the hadron degree of freedom rather than

¹ The article is published in the original.

that of quarks. At small x , the structure functions is prevailed by the gluon contribution, gluons with tiny momenta that identified with the Pomeron in QCD. The pomeron is a Regge trajectory, a family of particles with increasing spin, postulated to explain the slowly rising cross section of hadronic collisions at high energies. In the Regge theory, the diffractive forward scattering amplitude is described by the swap of the vacuum quantum number, which is so called Pomeron. The holographic description of QCD is a tool to calculate the non-perturbative quantities in QCD. We can use the AdS/CFT correspondence that links the strong coupling gauge theory at the boundary to the classical theory of the gravitation in the bulk AdS space [1]. Using AdS/CFT correspondence, we can apply our analysis of the strong coupling gauge theory in the usual Minkowski space by the classical theory of gravitation in the higher dimensional curved space. With respect to this correspondence we have two different manner for description of QCD, top-down approach which originates from the string theory in the higher dimensional space [2] and another one is bottom-up approach that is more phenomenological [3]. For Deep inelastic scattering description Brower, Polchinski, Strassler, and Tan (BPST) performed string calculations for the structure functions [4]. They suggest a kernel that gives contribution of pomeron swap to cross sections, with respect to this kernel one can explain two-body scattering amplitude. The BPST kernel describes the Pomeron (graviton) exchange contribution to the total cross section. Their study results are consistent with experimental data of structure functions only if one use a super local approximation [5]. The BPST kernel is for conformal field theory and is not appropriate for QCD studies there fore we must use modified BPST kernel [6].

2. THE MODEL

We concentrate on the two-body scattering for a process $1 + 2 \rightarrow 3 + 4$, at the high energy. Amplitude of this process is given by [5]:

$$A(s, t) = 2is \int d^2b e^{ik_{\perp} \cdot b} \int dz dz' P_{13}(z) P_{24}(z'), \quad (1)$$

where s is the invariant mass square $s = (p_1 + p_2)^2$ and t is the momentum transfer $t = (p_1 - p_2)^2$. For the near-forward scattering with a condition $s \gg t$. This amplitude is valid and dominated by the pomeron swap. The impact parameter b is the transverse vector perpendicular to the forward direction. The eikonal form $1 - e^{i\chi(s, b, z, z')}$, represent the BPST pomeron exchange kernel. $P_{13}(z)$ and $P_{24}(z')$ are overlap functions of incoming and outgoing particles with their 5D-coordinates z and z' . Physically, the overlap functions stand for the density distribution functions of participants in the AdS space. For deep inelastic scattering it is enough to consider the virtual photon $\gamma^*(1 = 3 = \gamma^*)$ with the virtuality Q^2 and the nucleon ($2 = 4 = N$) in

the forward limit $t = 0$. Considering the γ^*N forward scattering amplitude With respect to the optical theorem we can determine the structure function.

For single pomeron swap the structure functions at small x are written as:

$$F_i(x, Q^2) = \frac{Q^2}{2\pi^2} \int d^2b \int dz dz' P_{13}^{(i)}(z, Q^2) \times P_{24}(z') \text{Im} \chi(s, b, z, z'), \quad (2)$$

where $P_{13}(z, Q^2)$ and $P_{24}(z')$ have information about internal structure of involved particles in scattering. $i = 2$ is for F_2 structure function and $i = L$ is for the longitudinal structure function. In the conformal limit, the analytical form of the imaginary part of $\chi(s, b, z, z')$ can be obtained, and the impact parameter integration can be demonstrated explicitly. $P_{13}^{(2)}(z)$ describes the longitudinally polarized photon, while $P_{13}^{(L)}(z)$ contains both transverse and longitudinal components [7]. $P_{13}^{(i)}(z)$ and $P_{24}(z')$ are expressed as:

$$P_{13}^{(2)}(z, Q^2) = Q^2 z (K_0^2(Qz) + K_1^2(Qz)), \quad (3)$$

$$P_{13}^{(L)}(z, Q^2) = Q^2 z K_0^2(Qz), \quad (4)$$

$$P_{24}(z') = \frac{e^{-k^2 z'^2}}{2z'^{2M}} (\psi_R^2(z') + \psi_L^2(z')), \quad (5)$$

where K_0 and K_1 are the modified Bessel functions of the second kind and $\psi_{R,L}^2(z')$ are normalizable modes for the bulk-to-boundary propagators. With the choice of the model parameters value of the conformal mass $M = 3/2$ by the analysis of the electromagnetic form factors of the nucleon at the large momentum transfer. also the soft-wall parameter $\kappa = 0.350$ [GeV] which could reproduce the masses of the proton and ρ meson simultaneously [8]. In this paper, we use these values to evaluate the overlap function. Eq. (2) can be rewritten as:

$$F_i(x, Q^2) = \frac{g_0^2 \rho^{3/2} Q^2}{32\pi^{5/2}} \int dz dz' P_{13}^{(i)}(z, Q^2) \times P_{24}(z') (zz') \text{Im}[\chi(s, z, z')], \quad (6)$$

where

$$\text{Im}[\chi(s, z, z')] = e^{(1-\rho)\tau} e^{-\left(\left(\log^2 \frac{z}{z'}\right)/\rho\tau\right)} / \tau^{1/2}, \quad (7)$$

$$\tau = \log(\rho z z' s / 2). \quad (8)$$

That g_0^2 and ρ are adjustable parameters of the model and s is the usual Mandelstam variable, i.e., $x \approx Q^2/s$. The BPST kernel in Eq. (6) is a conformal kernel with maximum symmetry. In QCD conformal symmetry is broken because of color confinement.

There are hard-wall and soft-wall models in the bottom-up approach. The hard-wall model used a sharp cut-off in the extra dimension to realize the confinement and chiral symmetry breaking, while the structure functions of proton in this model cannot match the experimental data well. In the soft-wall