INTRODUCTION

The exothermic chemical interaction of porous media with active gaseous reactants leading to the formation of condensed products is used in the high-temperature synthesis of inorganic materials. At normal pressure, the concentration of intrapore gas is too low to ensure a complete stoichiometric conversion of the initial condensed material; therefore, the pressure in the reaction zone decreases and additional amounts of gaseous reactant should be introduced by means of filtration from the ambient medium. The high-temperature process of conversion (combustion) in the regime of filtration transport of gaseous reactant became known as filtration combustion.

Filtration combustion is a complex set of interrelated processes of heat transfer, gaseous reactant transport, and heterogeneous chemical transformation. Therefore, a mathematical description of filtration combustion involves a complex system of multidimensional differential equations. In recent years, much effort has been put into a simplified description of separate elements of filtration combustion in order to gain deeper insights into the dynamics and structure of the filtration combustion front. The focus has been on modeling one-dimensional steady-state regimes of filtration combustion [1], when the thermal wave front propagates under conditions of steady-state filtration gas supply to it through the unburnt material or the combustion products. The problem of stability of the filtration combustion front in such a steady-state regime was examined in [2].

The authors of [3] studied unsteady regimes of filtration combustion within the framework of a three-dimensional mathematical model capable of describing the interaction of porous samples of various geometries with gases. It was demonstrated that the exothermic transformation front propagating through samples with a relatively small cross section and a high permeability and thermal conductivity exhibits a number of peculiarities (Fig. 1; along with thin flat objects, thin-walled tubular systems can be considered). A virtually constant pressure was maintained inside the sample pores, whereas the rate of gas supply from outside was controlled by the rate of chemical transformation, with the gas being supplied immediately into the reaction zone. Based on the known regularities of combustion of such systems, we used here analytical methods to examine the stability of the front of steady filtration combustion of porous materials with high permeability and thermal conductivity under the assumption that the intrapore pressure is constant (isobaric approximation). To make this assumption viable, it is

One-Dimensional Model of Steady Filtration Combustion Front

P. M. Krishenik and K. G. Shkadinskii

a Institute of Structural Macrokinetics and Problems of Materials Science, Russian Academy of Sciences, Chernogolovka, Moscow oblast, 142432 Russia

b Institute of Problems of Chemical Physics, Russian Academy of Sciences, Chernogolovka, Moscow oblast, 142432 Russia

e-mail: krishenik@rambler.ru

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Abstract—The stability of the front of a steady combustion wave propagating through a porous medium was examined. It was assumed that the inflow of the gaseous reactant from outside into the reaction zone is determined by the chemical interaction mechanism. Changes in the mass of the products and in the thermophysical parameters of the condensed component during combustion were taken into account. The wave structure arising during the exothermic conversion of such systems was studied, the boundary of steady combustion was determined, and the character of loss of stability with respect to one-dimensional spatial perturbations was examined.

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worthwhile to consider cylindrical porous samples with anisotropic permeability (with the radial permeability being higher than the axial one). In this case, it is possible to suggest a straightforward one-dimensional description of the filtration combustion front with a simplified mode of transport of gaseous reactant from outside, which is entirely controlled by the reaction mechanism in the absence of filtration resistance. At the same time, this model takes into account the influx of additional mass from outside and, hence, a change in the mass of the initial porous material. It is known [4] that a change of the thermophysical parameters of the wave transformation of gaseous system influences the structure and stability of the front.

**MATHEMATICAL MODEL**

In the coordinate system fixed to the front, the system of conservation equations for describing the combustion of porous materials reads as

\[
\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho w^2) = \frac{\partial}{\partial x}(\rho \alpha (1-m)(c_v + v c \eta) T) + \text{div}(\rho \text{pc} T w)
\]

\[
= \lambda \text{grad}(T)
\]

\[
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(\alpha v (1-m)(c_v + v c \eta) T) = \text{div}(\rho \alpha (1-m)(c_v + v c \eta) T)
\]

\[
= \rho \alpha v (1-m)(c_v + v c \eta) T
\]

\[
w = k_1 \text{grad}(p),
\]

where \( w = (w_x, w_y, w_z) \) is the gas velocity vector; \( m \) is porosity (for the sake of simplicity, set constant); \( T \) is the temperature of the medium; \( \nu \) is the parameter that characterizes changes of the condensed component during exothermic conversion; \( \eta \) is the extent of conversion; \( c_v, \rho, \) and \( \mu \) are the heat capacity, density, and molecular mass of the gaseous component; \( c_t, \rho_t, \) and \( \mu_t \) are the heat capacity, density, and molecular mass of the gaseous component; \( Q \) is the heat effect of the chemical reaction; \( k \) is the preexponential factor; \( E \) is the activation energy; \( R \) is the universal gas constant; \( x \) and \( t \) are the space coordinate and time; \( \lambda = \lambda(\eta) \) is the effective conductivity of the condensed component; and \( T_0 \) is the initial temperature.

The porous medium is considered to be involved in continuous heat and mass exchange with the ambient gaseous medium. Taking into account the peculiarities of the reactive porous medium (quasi-two-dimensional character), we simplified the gas velocity vector to \( w = (0, 0, w_z) \), a form that describes only the predominant filtration gas supply in the transversal direction. Moreover, we used the isobaric approximation in describing the process of filtration; i.e., pressure variations are considered small. Mass transfer is largely governed by intense gaseous reactant consumption in the reaction zone with the formation of a condensed combustion product. Note that the mass of gas consumed is comparable with the mass of reacted condensed medium. A relatively weak mass transfer occurs in the preheat zone due to the expansion of the gas. At moderate initial pressures, this transfer can be safely neglected, even more so that the gas outflowing from the porous medium causes no changes in the temperature of the composition. On the contrary, the gas inflowing into the reaction zone should be heated from the initial temperature to the temperature of the products. That the masses of the condensed material and inflowing gaseous reactant are comparable requires that this heat exchange should be taken into account in the overall heat balance of the combustion wave. Using boundary conditions of and steady-state approximation for mass transfer,

\[
\lambda T' = \rho w_z mc(T - T_0),
\]

\[
mp w_z = \rho \alpha v (1-m)(\int \eta' + u \eta_d') dz,
\]

and averaging the system of Eqs. (1)–(5) over the transverse direction \( z \), we obtained a simplified mathematical model of filtration combustion front propagation. For the sake of convenience, we will retain the same notations for the averaged temperature and extent of conversion:

\[
\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho w^2) = \frac{\partial}{\partial x}(\rho \alpha (1-m)(c_v + v c \eta) T) + \text{div}(\rho \alpha (1-m)(c_v + v c \eta) T)
\]

\[
= \rho \alpha v (1-m)(c_v + v c \eta) T
\]

\[
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(\alpha v (1-m)(c_v + v c \eta) T) = \text{div}(\rho \alpha (1-m)(c_v + v c \eta) T)
\]

\[
= \rho \alpha v (1-m)(c_v + v c \eta) T
\]

\[
w = k_1 \text{grad}(p),
\]

where \( \eta' = \int \eta' + u \eta_d' \), and \( \eta' \) is the extent of conversion.

Let us describe the regimes of uniform propagation of the flame in the form of a running wave: \( T(x, t) = T(x + ut) \) and \( \eta(x, t) = \eta(x + ut) \), where \( u \) is the front propagation velocity. The coordinate of the narrow reaction zone of the steady front is set to be located at \( x = 0 \). This mathematical problem is nonlinear and cannot be solved analytically. If the activation energy of the chemical reaction is high (\( \beta = RT/E \ll 1 \), where \( T_f \) the maximum temperature in the reaction zone)