Calculation Formulas of Linear Geometrical Spreading for Ray Tracing in a 3D Block Inhomogeneous Gradient Medium

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Abstract—Suitable recurrent formulas are derived for direct programming of linear geometrical spreading of a central field of seismic rays in a 3D block gradient medium. These formulas are needed to organize shooting for area observation systems. For recalculation formulas across an interface, a new representation which involves a special nonorthogonal projection operator is found. This operator allows for additive separation of terms that depend only on the ray curvature, boundary curvature, and variable character of the velocity ratio along the boundary. Formulas which express partial derivatives of the eikonal via linear and angular geometrical spreadings are presented.

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INTRODUCTION

Starting with [1, 2], the ray method has been widely used to numerically solve various problems in mathematical physics. In calculation schemes of the ray method, computation of geometrical spreading of a central ray field holds center stage. There are numerous works (see, e.g., [2–10]) which deal with questions of calculating geometrical spreadings in general two- and three-dimensional media with curvilinear interfaces. For blockwise constant and blockwise gradient media (in this paper, no distinction is made between the two), explicit expressions for the ray, eikonal, and geometrical spreading have been intensively employed in computer realizations of the ray method (see [11, 12]).

In this paper, we come up with an elementary derivation of explicit calculation formulas for linear and angular geometrical spreadings of seismic rays in isotropic 3D media with blockwise constant gradient and piecewise smooth interfaces. The formulas are represented in a form that can be used for direct programming, and have been tested numerically by comparison with calculations obtained by the finite difference method. Also, an expression for second derivatives of a single-point eikonal is given in terms of linear and angular spreadings (as distinct from [13, 14] where these derivatives are expressed via front curvatures).

In recent years, 3D seismic prospecting with area observation systems has been widely used in prospecting geophysics. A currently central problem, therefore, is developing an effective advanced software to solve 3D problems on kinematics and dynamics of 3D waves for a class of media that would be sufficiently large to meet practical needs. Here, the problem of ray tracing from a given source location, $M_0$, to a given set of points, $M := M_i$, $i = 1, 2, \ldots, N$, which form a 2D arrangement of receivers on an observation surface, $S$, is much more complicated in comparison to the 2D case.

When solving the problem of fast shooting, it is natural to use smoothness properties of projections of Cartesian coordinates $x(M)$, $y(M)$, and $z(M)$ of the point $M$ on the observation surface $S$ treated as functions of ray parameters $\theta$ and $\varphi$ specifying the polar and azimuthal angles with which a ray exits

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the source point \( M_0 \). To this end, it is required that calculation algorithms for area hodographs include procedures for computing partial derivatives of the vector function \( r(M) = (x(M), y(M), z(M)) \) with respect to parameters \( \theta \) and \( \varphi \). We call these partial derivatives \( \frac{\partial r(M(\theta, \varphi))}{\partial \theta} \) and \( \frac{\partial r(M(\theta, \varphi))}{\partial \varphi} \) the vectors of linear (polar and azimuthal) geometrical spreading over the surface \( S \). It should be noted that this terminology is not conventional in the literature on calculation formulas for the (3D) geometrical spreading

\[
J = \left[ \frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial \varphi} \right] \cdot \tau.
\]

Here \( \tau \) is the unit vector tangent to the ray at a point \( M \), and \( \frac{\partial r}{\partial \theta} = r_\theta \) and \( \frac{\partial r}{\partial \varphi} = r_\varphi \) are partial derivatives of the radius vector of the ray point \( M \) with respect to \( \theta \) and \( \varphi \) at fixed time \( t = t(M) \), which is a ray parameter.

Numerical implementation of the ray method in the 2D case was brought in sight in [2] where it is proposed to determine \( J \) via simultaneous numerical integration of a ray equation and variation equations for \( r_\theta \) and \( r_\varphi \). Later, this approach was generalized to the 3D case. However, until recently, due to insufficient capacity of computational facilities, the problem of area shooting has not even been raised, and researchers’ efforts were mainly directed toward decreasing the number of differential equations needed for finding \( J \) [3, 5, 7].

A shooting algorithm for area observation systems using quantities calculated for a medium with piecewise constant velocity is described in [15]. In fact, this algorithm is appropriate for models with arbitrary velocity structures.

1. MEDIUM MODEL. PROBLEM STATEMENT

We consider a 3D isotropic medium with block gradient propagation velocity of elastic waves. Here, the only important fact is that we deal with a ray that passes from one interface to another with a velocity that linearly depends on Cartesian coordinates \((x, y, z)\). Therefore, the question as to the global structure of a medium model—blockwise or layered—is insignificant. As usual, by a blockwise model we mean one that allows crossing of the media interfaces within the mathematical bar

\[
0 \leq x \leq A, \quad 0 \leq y \leq B, \quad 0 \leq z \leq H, \quad A, B, H > 0.
\]

A model is layered whenever interfaces are not closed and their edges contact the bar’s lateral boundary.

Let a perturbation source be located at a point \( M_0 \). Also, let \( L^* \) be a ray exiting \( M_0 \), meeting successively \( n - 1 \) interfaces \( S_{k_1}, \ldots, S_{k_{n-1}} \) at which reflection or refraction (with or without exchange) occurs, and entering a point \( M_s \) on an observation surface \( S \). A sequence \( k_1, \ldots, k_{n-1} \), together with a feature specifying the character of passing through interfaces (reflection or refraction) and the type of exchange (\( PP, SS, PS, \) or \( SP) \), is typically called the ray code of \( L^* \). Below, the sequence of interfaces is renumbered as follows: \( S_i, i = 1, 2, \ldots, n - 1 \).

Denote by \( t_s, \varphi_s, \) and \( \theta_s \) the values of ray parameters \((t, \varphi, \theta)\) at \( M_s \). A ray \( L^* \) is assumed to be regular (with respect to a given code) if, for any \((\varphi, \theta)\) sufficiently close to \((\varphi_s, \theta_s)\), there exists a ray \( L \) (with parameters in a source \((\varphi, \theta)\)) which enters a point on \( S \) with the same code as has the ray \( L^* \). For \( L^* \) to be regular, it suffices to require that there are no zero “sliding angles” of the ray with the interfaces and observation surface and that these surfaces are sufficiently smooth.

In what follows \( t, \varphi, \) and \( \theta \) are ray parameters of points \( M \) on rays \( L \) outgoing from \( M_0 \) and incoming to \( S \) with the same code as has \( L^* \). If necessary, these rays can be continued outside \( S \) along a ray in a medium in which they approached \( S \). This is important if \( S \) coincides with an interface. Let \( r(t, \varphi, \theta) \) be the radius vector of a point \( M(t, \varphi, \theta) \). Then the regularity conditions for the ray \( L^* \) imply that in a sufficiently small vicinity of the triplet \((t_s, \varphi_s, \theta_s)\), the function \( r(t, \varphi, \theta) \) is continuously differentiable sufficiently many times.

Along with the parameters \( t, \varphi, \) and \( \theta \), a triplet of parameters \( s, \varphi, \) and \( \theta \) are also used for points \( M \), where \( s \) is the length of the path passed by a perturbation from point \( M_0 \) to point \( M \) along the ray.