Simulation of Wave Processes in a Vapor-Liquid Medium

V. G. Gasenko1*, G. V. Demidov2, V. P. Il’in2**, and I. A. Shmakov2***

1Kutateladze Institute of Thermophysics, Siberian Branch, Russian Academy of Sciences, pr. Akad. Lavrent’eva 1, Novosibirsk, 630090 Russia
2Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch, Russian Academy of Sciences, pr. Akad. Lavrent’eva 6, Novosibirsk, 630090 Russia

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Abstract—Numerical methods for the simulation of nonlinear wave processes in a vapor-liquid medium with a model two-phase spherical symmetric cell, with a pressure jump at its external boundary are considered. The viscosity and compressibility of the liquid, as well as the space variation of pressure in the vapor, are neglected. The problem is described by the heat equations in the vapor and liquid, and by a system of ODEs for the velocity, pressure, and radius at the bubble boundary. The equations are discretized in space by an implicit finite-volume scheme on a dynamic adaptive grid with grid refinement near the bubble boundary. The total time derivative is approximated by a method of backward characteristics. “Nonlinear” iterations are implemented at each time step to provide a specified high accuracy. The results of numerical experiments are presented and discussed for the critical thermodynamic parameters of water, for some initial values of the bubble radius and pressure jump.

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1. INTRODUCTION

The evolution of nonlinear waves and heat-and-mass transfer in two-phase bubble media are of importance for chemical and oil and gas technologies, power reactors, utilization of ecologically harmful substances, etc. Extensive literature is devoted to these problems (see [1–7] and papers cited therein).

In this paper, a vapor-liquid homogenized medium is considered. The medium is a homogeneous incompressible liquid with density $\rho_l$, containing uniformly distributed spherical vapor bubbles of the same radius $R$. It is assumed that $R \ll d \ll L$, where $d$ is the average distance between neighboring bubbles, and $L$ is the typical size of a bounded calculation domain $\Omega$. In other words, the behavior of a two-phase monodisperse mixture with a rather small volume concentration of the disperse phase is investigated. The effects of viscosity, chaotic motion, rotation, and deformation of bubbles, as well as their direct interactions and collisions, are neglected. The processes of fragmentation, coagulation, and formation of new disperse particles are assumed to be absent.

The vapor is considered an ideal gas whose parameters are interrelated by the equation of state

$$p_v = B \rho_v T,$$

where $p_v$, $\rho_v$, and $T$ are the pressure, density, and temperature (in degrees Kelvin) in the bubble, and $B = 461.5 \text{J/(kg-K)}$ is the specific gas constant for water vapor.

*E-mail: gasenko@itp.nsc.ru
**E-mail: ilin@scc.ru
***E-mail: i_shmakov@ngs.ru
The one-dimensional equations of conservation of mass and momentum of a homogeneous gas or liquid medium in a spherical system of coordinates have the following form:

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho u r^2) = 0, \\
\frac{\partial (\rho u)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho u^2 r^2) + \frac{\partial p}{\partial r} = 0,
\]

where \(u\), \(p\), and \(\rho\) are the velocity, pressure, and density. Viscosity is assumed to be absent.

The temperature distribution in the vapor bubble is described by the heat conduction equation in a spherical system of coordinates

\[
c_v \rho_v \frac{dT}{dt} \equiv c_v \rho_v \left( \frac{\partial T}{\partial t} + u_v \frac{\partial T}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \lambda_v r^2 \frac{\partial T}{\partial r} \right) + \frac{dp_v}{dt}, \quad 0 < r < R,
\]

and satisfies the initial and boundary conditions

\[
T(r, 0) = T^0(r), \quad \frac{\partial T}{\partial r} \bigg|_{r=0} = 0, \quad T \bigg|_{r=R} = T_s(p_v),
\]

where \(c_v\), \(\lambda_v\), and \(u_v\) are the heat capacity of vapor at constant pressure, heat conductivity, and motion speed, respectively, and \(T_s(p_v)\) is the saturated vapor temperature at pressure \(p_v\), which is assumed to be independent of \(r\) in the bubble, that is, \(\partial p_v/\partial r = 0\) (homobaric approximation under the assumption \(p_v = p_l \mid_{r=R}\)). The vapor density \(\rho_v\) in (3) is found from the equation of state (1). Here and below, the mathematical statement is that of [1, 2].

In the liquid, the initial boundary value problem for the temperature is as follows:

\[
c_l \rho_l \frac{dT}{dt} \equiv c_l \rho_l \left( \frac{\partial T}{\partial t} + u_l \frac{\partial T}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \lambda_l r^2 \frac{\partial T}{\partial r} \right), \quad R < r < R_\infty < \infty,
\]

\[
T(r, 0) = T^0(r), \quad T \bigg|_{r=R} = T_s, \quad \frac{\partial T}{\partial r} \bigg|_{R_\infty} = 0,
\]

where \(c_l\) and \(\lambda_l\) are the heat capacity and heat conductivity of the liquid, respectively. In Eqs. (2)–(4), a cell model is used. This model is based on the assumption that in the domain \(0 < r < R_\infty < \infty\) the fields of velocities and pressures are those for an isolated bubble in an unbounded domain with pressure \(p_\infty\) at infinity. In this statement, it is assumed that \(p_\infty\) is an instantaneously applied pressure, which is constant for \(r = R_\infty\). The heat flux is set zero at the cell boundary \(r = R_\infty\), which follows from the principle that the temperature is in equilibrium in the entire liquid (an adiabatic cell model). A fixed temperature, \(T^0(R_\infty)\), at the cell boundary can also be assumed. This means that there is heat inflow or outflow. The heat capacity and heat conductivity versus temperature in (3), (4) can be determined with either empirical formulas or tabulated experimental data.

Solving the “internal” and “external” temperature problems, one can determine the condensate mass flux \(j\) on the bubble surface

\[
j = (q_{v,R} - q_{l,R})/q, \quad q_{v,R} = -\lambda_v \frac{\partial T}{\partial r} \bigg|_{R^-}, \quad q_{l,R} = -\lambda_l \frac{\partial T}{\partial r} \bigg|_{R^+},
\]

where the heat flux values are taken for \(r < R\) and \(r > R\), respectively, and \(q = 2.26 \cdot 10^6\) J/kg is the specific heat of water evaporation. If \(j\) is positive, it denotes the mass flux of the evaporated liquid. Otherwise, it is the mass flux of the condensed vapor. Notice that in (5), \(r'\) is a Lagrangian radial coordinate moving with the bubble boundary \(R(t)\).

The liquid and vapor velocities at the bubble boundary, \(U_l\) and \(U_v\), differ from the interface velocity \(\dot{R} = \frac{dR}{dt}\) by a quantity determined by the condensate mass flux:

\[
U_l \equiv u_l \mid_{R} = \dot{R} - \frac{j}{\rho_l}, \quad U_v \equiv u_v \mid_{R} = \dot{R} - \frac{j}{\rho_v}.
\]