INTRODUCTION

The study of the flow of liquids in capillaries of a fiber system is a topical problem in the development of optimum modes of impregnation with a binding agent. The ability of a composite to provide the designed physicomechanical properties, hardness and toughness in different calculation cases, is established at the impregnation stage. This leads to the necessity to develop a mathematical model of hydrodynamics of liquids, including binding agents used during the production of composite materials. It is obvious that its specificity is due to the scaling effects.

In [1], original experimental data are presented which make it possible to mention the existence of a series of nonclassical scaling effects revealed when studying the hydrodynamics of liquid in a capillary porous medium on the microlevel.

In [2], an attempt at explaining the effects mentioned above within the model derived from the solution of the classical Navier–Stokes equations was implemented. The comparison of the derived solutions with the experimental data led to the following conclusions.

1. The solution describes the presence of the “starting” pressure both for wetting and for nonwetting liquids in capillary systems.

2. The solution describes the phenomenon of impregnation in the absence of a pressure difference and a constant impregnation rate; however, the mirror of the wetting liquid always remains flat independent of the properties of the fiber–liquid adhesion pair.

Thus, the existence of the meniscus of the liquid in the wetting mode (at zero pressure difference) within the classical formulation of the Navier–Stokes equations could not be described.

3. The solution cannot be related to the capillary pressure according to Laplace, since in accordance with it the mirror of the liquid in the wetting mode has no curvature.

4. The solution in the filtering mode makes it possible to select two layers: boundary turbulent and axial lamellar ones in the flow of liquid. However, additional physical parameters which could be related to the critical pressure difference \( P_1 \), upon the attainment of which the fluid flow rate increases sharply, are absent in the formulation of the Navier–Stokes equations.

As a result, it was concluded that it is necessary to formulate a generalization of the Navier–Stokes equations for the description of the experimental effects studied in [1], which cannot be explained within the classical formulation.

FORMULATION AND SOLUTION

OF THE PROBLEM OF THE FLOW
OF LIQUID IN A CAPILLARY POROUS
MEDIUM ON THE BASIS
OF THE GENERALIZATION
OF NAVIER–STOKES EQUATIONS

Let us consider the problem of the flow of liquid in the capillary porous structure of a unidirectional fiber material at the pressure difference \( P \). Let \( l \) be the
length of the capillary/fiber and \( \eta, 2H \), and \( r \) be the dynamic viscosity, average statistical distance between fibers, and the fiber radius, respectively.

The same as in [2], let us introduce the following hypotheses (assumptions).

1. Fibers are parallel to each other.
2. Packing of fibers is tetragonal; i.e., the axes of the fibers are located in the corners of the square net with the side \( 2H \). It follows that the relative volumetric fraction of the fibers \( \varphi \) can be calculated using the relation \( \varphi = (\pi r^2)/(4H^2) \). Accordingly, the distance between fibers can be calculated in terms of the relative volumetric fraction and the fiber radius \( H = r\sqrt{\pi/\varphi}/2 \).
3. The periodicity cell containing the isolated fiber surrounded by liquid is replaced by the equivalent layered cell as shown in the figure.

The fiber is “smeared” into two half-layers with the thickness \( t = H\varphi = r\sqrt{\pi/\varphi}/2 \) and width \( 2H \). The gap \( 2h \) between half-layers is \( h = H - t = H(1 - \varphi) = r \times \sqrt{\pi/\varphi} (1 - \varphi)/2 \).

4. The velocity field \( v \) has only one (axial) component \( v \). Moreover, \( v \) is the function of two coordinates \( x, y \).
5. The liquid is incompressible, i.e., \( \partial v_i/\partial x_i = 0 \). It follows that \( v(x, y) = v(y) \).

In accordance with the above hypotheses, the Navier–Stokes equations are reduced to one common differential equation with respect to the single components of the velocity of liquid \( v = v(y) \):

\[
\eta v'' + P = 0,
\]

where \( P \) is the pressure difference over the length \( l \) with dimensions [Pa/m].

This model was used in [2] and requires generalization. Here we propose the generalization of the classical model (1) of the following form:

\[
\eta v'' - \frac{\eta}{h_m^2} v + P = 0.
\]

The introduced term can be treated as the pressure difference due to the resistance force proportional to the velocity \( v \). The proportionality coefficient \( \eta/h_m^2 \) is expressed in terms of the nonclassical physical parameter \( h_m \) with dimensions of length. It is easy to see that Eq. (1) is the limiting case of Eq. (2) at \( h_m \to \infty \). When the \( h_m \) value is limited from above, the solutions of Eq. (2) will differ from the solutions of Eq. (1) the greater, the less the \( h_m \) value. On the other hand, if rather wide channels of the flow are considered, for which the value \( h_m \) \( v'' \) can be ignored in comparison with the flow velocity \( v \), relation (2) takes the form of Darcy’s law:

\[
\eta v = K P,
\]

where \( K = h_m^2 \) is the filtration coefficient in Darcy’s law.

The general solution of problem (2) with allowance for the symmetry over the coordinate \( y \) has the form

\[
v(y) = \frac{P h_m^2}{\eta} + C \frac{\cosh(y/h_m)}{\cosh(h/h_m)}.  \tag{3}
\]

Let us formulate the boundary value problem in the most general form:

\[
\eta v' + k(v - v_w) = 0,  \tag{4}
\]

where \( k \) is the friction coefficient between the liquid and the surface of the capillary porous medium. At \( k \to 0 \), the liquid “slips” along the fibers \( \tau(h) = \eta v' h \to 0 \) and hardly interacts with the surface; at \( k \to \infty \), the liquid wets the surface of the fibers \( v(h) \to v_w \). The impregnation rate is a physical characteristic of the surface of the “capillary porous medium–liquid” system and is defined as the impregnation rate in the absence of a pressure difference.

Substituting (3) into (4), we determine the only integration constant:

\[
C = \left( \frac{v_w - \frac{P h_m^2}{\eta}}{1 + \frac{\eta}{kh_m} \tanh(h/h_m)} \right)^{-1}.
\]

We define the average flow rate of liquid \( \bar{v} \) proportional to the fluid flow rate per second:

\[
\bar{v} = \frac{1}{2h} \int_{-h}^{+h} v(y) dy = v_w + \left( \frac{P h_m^2}{\eta} - v_w \right) \times \left\{ 1 - \frac{1}{1 + \frac{\eta}{kh_m} \tanh(h/h_m) \tanh(h/h_m)} \right\}. \tag{5}
\]