INTRODUCTION

The role of biodiversity in the enhancement of ecosystem stability has been studied for a relatively short time. Until recently, studies in this field have been confined to the accumulation of data on the biodiversity of soil and vegetation communities and their mutual successions. The attention of cyberneticists was drawn to the fundamental role of biodiversity in the system theory when the so-called law of requisite variety was formulated by C. Shannon and W. Ashby. In ecology, this law explains the stability associated with the arrival of energy and mass into the system from the environment and the internal redistribution of biological resources. An understanding of the significance of biological diversity resulted in awareness of the need to conserve diversity for sustainable development and rational nature management.

A large number of general studies that relate to ecosystems and the biosphere are devoted to sustainable development. It has been claimed (Pegov, 2004) that the Earth’s natural system is currently approaching the bifurcation point, behind which development can occur via several phase trajectories with unequal probabilities. When passing to a new stable state, the adaptation of the systems of different biological complexities is qualitatively identical. The synergetic unity of the descriptions of these systems has allowed researchers to distinguish and predict the tendencies of their development based on the known examples.

In the Russian Federation, the concept of sustainable development enables the practical implementation of theoretical postulates; however, at the time of this writing, the development of these postulates is far from perfect. This concept is based on limiting the consumption of biosphere production (Dobrovolskii, 1997). This limit is determined by the biotic regulation theory and equal to approximately 1% of the total biosphere production. The limit was first surpassed in the second half of the 20th century.

Let us digress from the general and global problems and consider soil–vegetation systems (SVSs), which are represented by pasture lands in order to pay closer attention to bifurcation processes that currently occur all over the world at all levels of biosystem organization. The dynamics of the desertification of pasture lands on black earth soils in the Republic of Kalmykia, which are represented by a vast territory of sandy loam and sandy soils with an area of more than 2 million hectares, are based on satellite monitoring data. This territory has been used for years as a highly productive distant winter pasture. Soil deflation processes were traced according to satellite images (SIs) recorded in 1954–1995 at a spatial resolution of 5–50 m. SIs are valuable materials for simulating economic and environmental effects of anthropogenic impact on soil ecosystems of black earth soils.

We studied the SVS degradation due to pasture digression, which reduces the phytomass recovery (Salugin, 2001). In this context, the question relating to the dynamic stability of these vegetation systems is particularly topical, since the identification of the reasons for the disturbance of pasture self-recovery can underlie the general theory of ecosystem sustainability, whereas the numerical determination of the model parameters will make it possible to apply this theory.

On one hand, these systems are vital in various aspects (conservation of sustainable landscape, biosphere reproduction, prevention of agriculture and cattle breeding collapses, urban development, etc.); one the other hand, they are used as reference systems for natural zones under study. Therefore, the designed methods can be extrapolated on other areas both in...
Russia and abroad. Oil and gas deposits and agriculture have been intensively developed in the southeastern region of European Russia. The situation is aggravated by natural climatic conditions. Almost the entire ecosystem of the pasture in the region has degraded because of the anthropogenic impact. Environmental catastrophes occurred in certain areas, where the deflation was of an avalanche-like character. The acute problem of pasture recovery emerged in the former Soviet Union.

When studying soil systems of the black-earth region, a paradigm of multichannel transitions (successions) between elements was developed via the Markov-chain (MC) mechanism. The dynamics of the degradation and recovery of classes of ecosystems in the black-earth region was studied in approximation of uniform Markov processes using the MC theory (Kulik, 2003). The ergodicity of MCs for long-term prediction of ecosystem condition was determined under this approximation. The resulting ergodicity predetermined their efficiency for forecast simulation. A comparison of the stationary distributions of classes of the systems derived from higher-order matrices made it possible to ascertain that the process is nonlinear, as well as to determine the bifurcation points. The possibility of determining the time of the stable existence of classes was demonstrated.

When performing the mathematical modeling of successions in pasture ecosystems using MCs, we used transition matrices that describe the destruction and recovery processes. One of the problems of constructing an adequate MC was associated with ensuring the required accuracy (−2%) for plotting a weighed focused count of the model.

METHODS

The mathematical experiment was based on a theoretical modeling of dynamic transitions using ordinal differential equations (ODEs). The parametric stability of SVS was discussed in terms of Lyapunov stability (Petrov, 2004). The investigation procedure was based on calculation experiments that represent the dynamics of the processes in systems with different degrees of soil cover degradation.

In this study, mathematical modeling was used to investigate the stability of SVS as a result of stability tests of an ODE system; we focused mainly on the parametric stability.

The resolution of the system of ODEs is stable if the resolution using a slightly changed initial condition is close to the initial resolution. The system’s ability to conserve its stability at variations in the parameters is known as parametric stability.

Let us distinguish between the initial values \( \alpha_{ij}^{ini} \) of parameters of the mathematical model of the system and the varied values as follows:

\[
\alpha_{ij}^{var} = \alpha_{ij}^{ini}(1 + \varepsilon_{ij}),
\]

where \( \varepsilon_{ij} \) are numbers that are small compared to unity and can be either positive or negative. Values \( \varepsilon_{ij} \) are usually unknown and correspond to the error in measuring \( \alpha_{ij}^{ini} \) parameters or their variation with time.

The fact that model variations are determined using Eq. (1) implies that the relative effect of parameter variation rather than absolute effect is studied. In the present study, we consider linear systems of ODEs with coefficients for which the initial values remain constant. In this case, solutions are either asymptotically stable (i.e., \( s_i(t) \to 0 \) at \( t \to \infty \)) or unstable. For nonlinear systems, there can be a stable solution for certain initial conditions and an unstable solution for the other systems; thus, stability is studied for a particular case. In a linear model, an investigation of the stability can be extrapolated on the entire class of the systems being simulated. In the mathematical model, an additional variable is used to describe the management as an external effect. The characteristic polynomial and its roots are subsequently determined. If the real components of all of the roots are negative, the system is stable.

The fact that the stability theory is frequently used for the formal management of complex systems stimulated our further research relating to ecosystem management aimed at the optimal utilization of SVS biological resources.

There has recently been a significant advance in the ODE method for ecosystem modeling due to the development of numerical methods and due to the fact that various calculation and simulation experiments have been developed and implemented. In our case, the studies of SVS stability were based on the conventional methods for determining the stability of ODE systems using the roots of the characteristic polynomial. Let us note that the test plot of the black-earth region in Kalmykia was represented by the following four pasture types:

- \( S_1 \), i.e., gramineous–kochia–white wormwood pastures with dense or relatively dense canopy;
- \( S_2 \), i.e., moderately and strongly rarefied gramineous, white wormwood, and \( Stipa capillata \) pastures;
- \( S_3 \), i.e., strongly rarefied \( Stipa capillata–Ceratocarpus arenarius \) L. and annual-weed pastures;
- \( S_4 \), i.e., drift sands (the final degradation stage).

These four types of pastures can be converted into each other, degraded, and recovered with different degrees of intensity (Salugin, 2001).

RESULTS AND DISCUSSION

The mathematical model of successions of black earth soils was described using a system of ODE with constant coefficients. This, so-called autonomous system, in which the right-hand side equation parts do not contain time variable, has been studied well and