Saturation of nuclear matter and realistic interactions

Kh. Gad

Physics Department, Faculty of Science, South Valley University, Sohag, Egypt

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Abstract. In this communication we study symmetric nuclear matter for the Brueckner-Hartree-Fock approach, using two realistic nucleon-nucleon interactions (CD-Bonn and Bonn C). The single-particle energy is calculated self-consistently from the real on-shell self-energy. The relation between different expressions for the pressure is studied in cold nuclear matter. For best calculations the self-energy is calculated with the inclusion of hole-hole (hh) propagation. The effects of hh contributions and a self-consistent treatment within the framework of the Green function approach are investigated. Using two different methods, namely, G-matrix and bare potential, the hh term is calculated. We found that using G-matrix brought about non-negligible contribution to the self-energy, but this difference is very small and can be ignored if compared with the large contribution coming from particle-particle term. The contribution of the hh term leads to a repulsive contribution to the Fermi energy which increases with density. For extended Brueckner-Hartree-Fock approach the Fermi energy at the saturation point fulfills the Hugenholtz-Van Hove relation.

PACS. 21.65.+f Nuclear matter

1 Introduction

The evaluation of the saturation properties of nuclear matter from the basic nucleon-nucleon interaction has been extensively studied using the Brueckner-type resummation of ladder diagrams. The Brueckner-Hartree-Fock (BHF) definition of the self-energy has been extended to account for the effects of hh ladders in a perturbative way [1-4]. For a consistent treatment, however, one should treat the propagation of particle-particle (pp) and hh states in the in-medium scattering equation on the same footing. This turned out to be a rather ambitious aim. Starting from a single-particle propagator, which is characterized for each momentum \( k \) by one pole at the quasi-particle energy \( \varepsilon_{qp} \), only, the in-medium scattering reduces to the Galitskii-Feynman approach. If the hh part of the propagator is ignored, one obtains an equation for the ladder diagrams for the reducible two-particle Green function that corresponds to the Bethe-Goldstone equation. Using the complete Galitskii-Feynman propagator for nuclear matter at zero temperature with realistic NN interaction leads to the so-called pairing instability [5-7].

It has been argued [8] that it would be more natural to choose the propagator according to the Green function method, i.e. define the single-particle propagator with a single-particle energy which includes the real part of the self-energy as a single-particle potential for particle and hole states. This leads to a spectrum which is continuous at the Fermi momentum, which provided the name “continuous choice” for this approach. If, however, the single-particle potential, \( U = 0 \) is assumed above \( k_F \), then we have the conventional or gap choice. The continuous choice leads to an enhancement of the correlation effects in the medium and tends to predict larger binding energies for nuclear matter than the conventional choice. It is important to point out that, in the present communication we will focus our attention on the continuous choice. Also, we use exact Pauli operator to carry out our calculations.

Works using realistic interactions lead to a reasonable result for the saturation density and the binding energy at the saturation point. However, in violation of the Hugenholtz-Van Hove theorem, the resulting Fermi energy \( E_F \) at the saturation point is usually very different from the binding energy per particle \( E/N \). Improvement of the fulfillment of the Hugenholtz-Van Hove property with respect to the G-matrix approximation is observed when using the quasi-particle T-matrix approach, or correction from hh [9,10].

It is known that the exact theory [11-13] should fulfill certain thermodynamical relations. We shall consider in the present work the equivalence of two ways of calculating the pressure in a system at zero temperature:

\[
P = \rho^2 \frac{\partial(E/N)}{\partial\rho}
\]

(1)

\[
= \rho(E_F - E/N),
\]

(2)
where $\rho$ is the nuclear matter density. The relation between the Fermi energy and the binding energy at the saturation point is given by [11]

$$E_F = E/N.$$  \hspace{1cm} (3)

This equality of the Fermi energy and the binding energy per nucleon was derived by Weiskopf [14] on the basis of the independent particle model [15] which has been considered to be only a rough approximation. These relations are satisfied by the exact theory and can also be satisfied in a perturbative calculation to a given order of the expansion parameter. The realization of the above relation is very important, since it would give confidence to the single-particle properties obtained in the calculations.

Symmetric nuclear matter within the conserving, self-consistent $T$-matrix approximation has been studied [16–18]. This approach involves off-shell propagation of nucleons in the ladder diagrams. They found that the thermodynamic relations are well satisfied unlike for a $G$-matrix or a $T$-matrix approach using quasi-particle propagators in the ladder diagrams. Also they found that the full $T$-matrix and the $G$-matrix calculations give similar results for $E/N$ using CD-Bonn and Nijmegen potentials [18].

Baldo and collaborators [19] obtained for the Argonne $v_{14}$ potential [20] a saturation density corresponding to 1.565 fm$^{-3}$ with about the correct amount of binding. This corresponds to an overestimation of the empirical density by about 60% but appears completely consistent with corresponding variational calculations [21] for the same potential. Gent group [22,23] studied the saturation of nuclear matter using a fully self-consistent treatment of short range. They found that the resulting saturation densities are closer to the empirical result when compared with (continuous choice) Brueckner-Hartree-Fock values. Arguments for the dominance of short-range correlations in determining the nuclear-matter saturation density are presented. It was argued in ref. [22] that the repulsive effect due to the inclusion of the $l$-$h$ scattering might be able to shift the saturation point close to the empirical value.

Frick [24] studied the properties of symmetric nuclear matter at finite temperature using SCGF approach. He found that the Hugenholtz-Van Hove theorem is well fulfilled in the HF and the SCGF approximation, while it is badly violated in the BHF approach. He also gets on the saturation density in the SCGF approximation, $\rho_{sat} = 0.31$ fm$^{-3}$. This value is still almost twice as large as the empirical saturation density ($\rho_0 = 0.17 \pm 0.02$ fm$^{-3}$).

Our main aim of the present communication is to investigate the relation between the binding energy and the Fermi energy taking into account different expressions for the pressure in cold nuclear matter with the Brueckner scheme and extended BHF (EBHF) approach. Throughout this paper two realistic $NN$ interactions will be used and compared with other approaches. The present paper also contains further developments on the problem at hand. Finally, using two different ways, we shall calculate the self-energy for lh term.

### 2 Formalism

The integral equation for the Brueckner $G$-matrix is given by

$$\langle k_1k_2 | G(\omega) | k_3k_4 \rangle = \langle k_1k_2 | v | k_3k_4 \rangle + \Sigma_{k<k_F}(\langle k_1k_2 | v | k_3k_4 \rangle \times \frac{1 - \Theta_0(\omega)}{\omega - \varepsilon_{k_3} - \varepsilon_{k_4}})$$

$x(\omega)$, appearing in the kernel of eq. (4) enforces the scattered momenta to lie outside the Fermi sphere, and it is commonly referred to as the "Pauli operator". The $G$-matrix depends parametrically on the starting energy $\omega$. The standard Brueckner approximation for the nucleon self-energy, has the following expression:

$$\Sigma_{BHF}(k,\omega) = \Sigma_{HF}(k) + \Sigma_{2\pi}(k,\omega) = \Sigma_{k<k_F}(\langle kk' | G(\varepsilon_k + \varepsilon_k') | kk' \rangle \gamma),$$

where $\varepsilon_k$ is the self-consistent single-particle energy, label $A$ means antisymmetrization, $\Sigma_{HF}(k)$ is the Hartree-Fock self-energy (it is independent of energy) and $\Sigma_{2\pi}(k,\omega)$ is the self-energy for the two-particle one-hole (2p1h). In the BHF approach, (4) and (5) are solved self-consistently (for more information, see [3,25–29]).

The self-consistent Green-function (SCGF) approach differs in two main aspects from the BHF approximation. Firstly, within SCGF particles and holes are treated on an equal footing, whereas in BHF only intermediate particle states are included in the ladder diagrams. This aspect ensures thermodynamic consistency, e.g. the Fermi energy or chemical potential of the nucleons equals the binding energy at saturation (i.e. it fulfills the Hugenholtz-Van Hove theorem). In the low-density limit BHF and SCGF coincide. As the density increases the phase space for lh propagation is no longer negligible. Second, the SCGF generates realistic spectral functions, which are used to evaluate the effective interaction and the corresponding nucleon self-energy.

The contribution of the lh terms to the self-energy in a kind of perturbative way is given by [30,31]

$$\Sigma_{G}^{2\pi}(k,\omega) = \int_{k_F}^{\infty} d^3p_1 \int_0^{k_F} d^3h_1 d^3h_2$$

$$\langle k, p_1 | G | h_1, h_2 \rangle^2 \frac{\omega + \varepsilon_{p_1} - \varepsilon_{h_1} - \varepsilon_{h_2} - i\eta}{\omega + \varepsilon_{p_1} - \varepsilon_{h_1} - \varepsilon_{h_2} - i\eta},$$

which is graphically represented in fig. 1. If we take only the contribution coming from fig. 1b, in this case the self-energy for the lh term is given by

$$\Sigma_{G}^{2\pi}(k,\omega) = \int_{k_F}^{\infty} d^3p_1 \int_0^{k_F} d^3h_1 d^3h_2$$

$$\times \frac{\langle k, p_1 | v | h_1, h_2 \rangle^2}{\omega + \varepsilon_{p_1} - \varepsilon_{h_1} - \varepsilon_{h_2} - i\eta}.$$