Doubly heavy-quark baryon spectroscopy and semileptonic decay

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Received: 8 November 2006
Published online: 12 March 2007 — © Società Italiana di Fisica / Springer-Verlag 2007

Abstract. Working in the framework of a nonrelativistic quark model we evaluate the spectra and semileptonic decay widths for the ground state of doubly heavy $Z$ and $\Omega$ baryons. We solve the three-body problem using a variational ansatz made possible by the constraints imposed by heavy-quark spin symmetry. In order to check the dependence of our results on the inter-quark interaction, we have used five different quark-quark potentials which include Coulomb and hyperfine terms coming from one-gluon exchange, plus a confling term. Our results for the spectra are in good agreement with a previous calculation done using a Faddeev approach. For the semileptonic decay our results for the total decay widths are in good agreement with the ones obtained within a relativistic quark model in the quark-dequark approximation.


1 Introduction

Even though only recently the mass of a baryon with two heavy quarks has been measured experimentally [1], these systems have been being studied for more than a decade. Working with a system with two heavy quarks one can take advantage of the constraints imposed by heavy-quark spin symmetry (HQSS). This symmetry amounts to the decoupling of the heavy-quark spins in the infinity heavy-quark-mass limit. In that limit one can consider the total spin of the two heavy-quarks subsystem to be well defined. This result, that we shall assume to be valid for the actual heavy-quark masses, will simplify the solution of the three-body problem.

In this contribution we shall present results for masses and total semileptonic decay widths. We have also analyzed other static observables as well as form factors, differential decay widths and angular asymmetries of the weak decays. For a detailed account of the full calculation see ref. [2].

In table 1 we summarize the quantum numbers of the baryons considered in this study.

2 The model

Table 1. Quantum numbers of doubly heavy baryons analyzed in this study. $S$, $J^p$ are the strangeness and the spin parity of the baryon, $I$ is the isospin, and $S_0^a$ is the spin parity of the heavy degrees of freedom. l denotes a light u or d quark.

<table>
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<tr>
<th>Baryon</th>
<th>$S$</th>
<th>$J^p$</th>
<th>$I$</th>
<th>$S_0^a$</th>
<th>Quark content</th>
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<td>$\frac{1}{2}$</td>
<td>1+</td>
<td>ccl</td>
</tr>
<tr>
<td>$\Xi_{cc}^*$</td>
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</tr>
<tr>
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</table>

In this study, the centre of mass (CM) motion has been removed, the intrinsic Hamiltonian that describes the inner dynam-

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ics of the baryon is given by

\[ H^{\text{int}} = \sum_{j=1,2} H^{sp}_{j} + V_{h_i h_2} (r_1 - r_2, \text{spin}) - \frac{\nabla_1 \cdot \nabla_2}{m_q} \pm \mathcal{M}, \]

\[ H^{sp}_{j} = -\frac{\nabla_2^2}{2\mu_j} + V_{h_j q} (r_j, \text{spin}), \quad j = 1, 2, \tag{1} \]

where \( r_1, r_2 \) are the relative positions of the \( h_1, h_2 \) heavy quarks with respect to the light quark \( q \). \( \mathcal{M} = m_{h_1} + m_{h_2} + m_q \), \( \mu_j = (1/m_{h_j} + 1/m_q)^{-1} \) and \( \nabla_j = \partial/\partial r_j \). \( j = 1, 2 \).

\( V_{h_i q} \) and \( V_{h_j h_2} \) are the heavy-light and heavy-heavy interaction potentials. Note the presence of the Hughes-Eckart term that results from the separation of the CM motion.

For the quark-quark interaction we have considered five different phenomenological potentials, one suggested by Bhaduri and collaborators [3] (BD) and four suggested by B, Silvestre-Brac and C. Semay [4, 5] (AL1, AL2, AP1 and AP2). All of them include Coulomb and hyperfine terms coming from one-gluon exchange and a confining term, and differ in the form factor used for the hyperfine term, as a form factor in the one-gluon exchange Coulomb term or in the power of the confinement term. All free parameters had been adjusted to reproduce the light and heavy-light meson spectra. Details on the potentials can be found in refs. [3–5].

For the interactions considered, the total spin and internal orbital angular momentum commute with the intrinsic Hamiltonian, and thus are well-defined. In this work we will study the ground state of baryons with total angular momentum \( J = 1/2, 3/2 \) so we can assume the orbital angular momentum to be 0. This implies that the spatial wave function only depend on \( r_1, r_2 \) and \( r_1 - r_2 \).

We will also assume that taking the total spin of the heavy degrees of freedom to be well defined, as obtained in the infinite heavy-quark mass limit, is a good approximation. That will allow us to write the wave function in a simple way (see ref. [2] for details).

The spatial part of wave function will be determined using a variational method in which we will assume the following functional form:

\[ \psi^{B}_{h_i h_2} (r_1, r_2) = N F^{B}(r_{12}) \phi_{h_i q} (r_1) \phi_{h_2 q} (r_2), \tag{2} \]

where \( N \) is a normalization constant, \( \phi_{h_i q} \) is the S-wave ground-state wave function \( \varphi_{j}(r_j) \) of the single-particle Hamiltonian \( H^{sp}_{j} \) corrected at large distances:

\[ \phi_{h_i q} (r_j) = (1 + \alpha_j r_j) \varphi_{j} (r_j), \quad j = 1, 2. \tag{3} \]

The heavy-heavy Jastrow correlation function \( F^{B} \) will be given as a linear combination of Gaussians:

\[ F^{B}(r_{12}) = \sum_{j=1}^{4} a_{j} e^{-\delta_{j}^{2} (r_{12} + d_{j})^{2}}, \quad a_{1} = 1, \tag{4} \]

where \( \alpha_{i}, a_{i} \) \( i \neq 1, b_{i} \) and \( d_{i} \) are free variational parameters. The values that we get for the variational parameters are compiled in ref. [2].

We have also used the wave function obtained in this model to study different doubly \( B(1/2 \pm) \rightarrow B'(1/2 \pm) \) baryon semi leptonic decays involving a \( b \rightarrow e \) transition at the quark level. We have worked in the spectator approximation with only one-body currents.

The differential decay width reads

\[ d\Gamma = 8 |V_{cb}|^{2} m_{B} G_{F}^{2} \frac{d^{3} p' d^{3} k' d^{3} p d^{3} k}{(2\pi)^{6} E_{B}'^{2}(2\pi)^{3} E_{B}(2\pi)^{3} E_{B}'(2\pi)^{3}} \]

\[ \times \delta^{4}(p - p' - k - k') \mathcal{L}^{\alpha\beta}(k, k') \mathcal{H}_{\alpha\beta}(p, p'), \tag{5} \]

where \( |V_{cb}| \) is the modulus of the corresponding Cabibbo-Kobayashi-Maskawa matrix element, \( m_{B} \) is the mass of the final baryon, \( G_{F} \) is the Fermi decay constant, \( p, p' \), \( k \) and \( k' \) are the four-momenta of the initial baryon, final baryon, final anti-neutrino and final lepton, respectively, and \( \mathcal{L} \) and \( \mathcal{H} \) are the lepton and hadron tensors.

The lepton tensor is given as

\[ \mathcal{L}^{\alpha\beta}(k, k') = \delta^{\alpha\beta} \rho^{k_{\mu} k'_{\nu}} \delta^{\mu\nu} k_{\alpha} k_{\beta}, \tag{6} \]

where we use the convention \( \rho^{123} = -1 \), \( g^{\mu\nu} = (+, -, -, -) \).

The hadron tensor is given as

\[ \mathcal{H}_{\mu\nu}(p, p') = \]

\[ \frac{1}{2} \sum_{r, r'} \left\langle B', r' p' | \mathcal{T}(0) \gamma_{\mu} (I - \gamma_{5}) \Phi_{B}(0) | B, r p \right\rangle \]

\[ \times \left( B', r' p' | \mathcal{T}(0) \gamma_{\nu} (I - \gamma_{5}) \Phi_{B}(0) | B, r p \right)^{*}, \tag{7} \]

with \( |B, r p \rangle \) \( \langle B', r' p' | \) representing the initial (final) baryon with three-momentum \( p \) \( (p') \) and spin index \( r \) \( (r') \). The baryon states are normalized such that

\[ \langle r p | r' p' \rangle = (2\pi)^{3} (E(p)/m) \delta_{rr'} \delta^{3}(p - p'). \tag{8} \]

We compute the widths similarly as we did in ref. [6] for baryons with a heavy quark.

3 Results and discussion

The mass of the baryon is simply given by the expectation value of the intrinsic Hamiltonian. In table 2 we give our results for doubly heavy \( \Xi \) baryons, while in table 3 are the results for the doubly heavy \( \Omega \) ones. Our central values correspond to the results obtained using the AL1 potential, while the errors quoted take into account the variations found when using the other potentials. That also applies to the quoted results for ref. [4], obtained with the same interaction potentials but within a Faddeev approach. When comparison with this work is possible we find an excellent agreement between the two calculations. Besides we give predictions for states not considered in the study of ref. [4]. We also compare with other theoretical models. All calculations give similar results that vary within a few percent. From the experimental side only the mass of the \( \Xi_{c} \) has been measured. The experimental